# Linear Programming Applied to the Optimization of Blind Adaptive Antenna Arrays

Caroline A. D. Silva and Marcelo A. C. Fernandes

***Abstract*— Arrays of adaptive antennas are widely used in wireless communication systems, where the array coefficients are adjusted in real time in order to reduce the rate of data errors due to interferences. Adjustment of the coefficient can be performed using supervised or blind methods. In the case of the latter, it is also possible to improve the transmission capacity of the system. Nevertheless, the blind algorithms used to optimize the array coefficients employ nonconvex functions, which reduce their efficiency in the presence of local minima. Different to the solutions that have been suggested in the literature, this article presents a proposal for convex optimization, based on linear programming, applied to optimization of the coefficients associated with arrays of adaptive antennas. The work also presents a comparative performance evaluation of interior-point, active-set, and simplex methods applied in the optimization process. Simulation results for digital communication systems are presented using bit error rate performance curves.**

***Keywords*— Linear programming, adaptive antennas, blind algorithm, convex function.**

## I. Introduction

In digital wireless communication systems, the signals can be corrupted by various factors, notably thermal noise and multipath phenomena that cause selective fading. Thermal noise, modeled by the addition of random variables with a given probability distribution, can be minimized by the use of channel encoders that utilize redundancy symbols to reconstruct the transmitted signal. However, it is important to note that multipath phenomena, caused by diverse reflections of the signal during transmission, are not efficiently treated using channel encoders. Multipath phenomena are one of the main causes of intersymbol interference (ISI), an effect characterized by the overlap of symbols from the same source, in the domain of time. ISI limits the transmission capacity of the channel, and is one of the main problems in digital wireless communication systems. Various devices can be used in the reception process in order to minimize the effects of ISI, amongst which are schemes involving adaptive antenna arrays. These are adaptive spatial filters that aim to compensate the ISI in order to recover the transmitted signal. ISI is dynamic and changes according to the environment, so it is necessary to use efficient adaptive algorithms applied to the antenna array. In turn, these adaptive algorithms provide a convenient means of adjusting the coefficients of the antenna array (the parameters to be optimized) in order to attenuate the ISI.

Amongst the various types of adaptive algorithm, those belonging to the class known as blind algorithms [1] do not require the transmission of a reference signal to the receiver in order to calculate the coefficients associated with the array. Blind algorithms minimize the problem of ISI from observation of the output signal of the communication channel, and the fact that they do not utilize a reference signal increases the data transmission capacity. There are various techniques available for the blind optimization of antenna array coefficients [1], [2], amongst which one of the most widely used is the gradient descent optimization technique known as the constant modulus algorithm (CMA) [1]. Nonetheless, the CMA optimization function does not provide global convergence, showing undesirable local minima that can result in inefficient reduction of ISI [1].

In order to improve the performance of blind algorithms, given the problems of local minima, the work described in [3] proposes a technique for blind optimization of the coefficients, with global convergence, based on a convex cost function. Following guidance in relation to the convex cost function, presented in [3], the work described in [4] elaborates a linear programming (LP) methodology applied to blind adaptive temporal filters (also known as equalizers), as an alternative to the CMA algorithm. Finally, the work presented in [5] provides a refinement of the technique described in [4], with new analyses and proposing a new restriction function and other LP optimization algorithms applied to a blind adaptive temporal filter known as the blind linear equalizer based on linear programming (BLE-LP). It can be seen that the reported LP studies involving blind optimization algorithms have only addressed adaptive temporal filters. Therefore, different to the previous studies [3], [4], [6], [5], the paper presents a proposal for a blind adaptive spatial filter based on LP, which will be called the blind adaptive antenna array based on linear programming (BAAA-LP).

## II. Array of adaptive antennas

In the discrete baseband digital communication system with ISI, a source of information transmitting complex symbols, $a(k) = a^I(k) + ja^Q(k)$, belonging to a set, $A = \{a_0, \ldots, a_{M-1}\}$, of $M$ possible symbols. The symbols are transmitted with a period of $T_s$ seconds, and each symbol is represented by words of $b$ bits. $T_s$ is the symbol sampling period, or symbol interval. The variables $a^I(k)$ and $a^Q(k)$ are, respectively, the unidimensional phase and quadrature components that compose the bidimensional transmitted signal. The received signal, $u(k)$, can be expressed by

$$u(k) = \sum_{i=0}^{L-1} \rho_i(k) a(k - \tau_i(k)), \tag{1}$$

Caroline A. D. Silva and Marcelo A. C. Fernandes, Department of Computer Engineering and Automation, Federal University of Rio Grande do Norte, Natal-RN, Brazil, E-mails: carolads@gmail.com, mfernandes@dca.ufrn.br.

where $L$ is the number of paths of the channel, $\rho_i$ is the complex gain of the $i$-th path, and $\tau_i(k)$ is an integer representing the delay of the $i$-th path at instant $k$.

An array of adaptive antennas is usually implemented as a linear vector of equally spaced elements that can direct of the gain of the antenna in a given direction, while annulling others. It is proven that the maximum number of nulls is given by $N-1$, where $N$ is the number of antennas in the array. Figure 1 illustrates a linear array where the antennas are oriented along the $x$-axis, with spacing $\Delta x$. It is assumed that all the multipaths arrive at the array in the horizontal plane, with an angle of arrival (AOA) of $\theta$ in relation to the $y$-axis orthogonal to the $x$-axis. Each $n$-th antenna is weighted by the conjugate of a complex gain, $w_n^*$, and the spacing $\Delta x$ should generally be greater than or equal to $\lambda/2$ ($\lambda$ is the wavelength of the transmitted signal) [7].

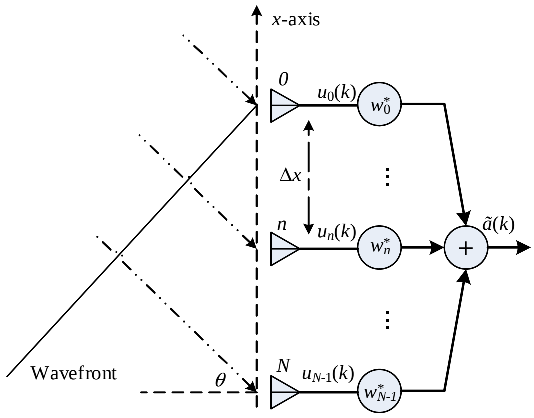



Fig. 1. Structure of a linear vector of $N$ equally spaced antennas.

The signal, $u(k)$, received on the $n$-th antenna element is given by

$$u_n(k) = u(k)e^{-j\beta n\Delta x\cos(\theta)} = u(k)v(\theta), \quad (2)$$

where $\beta = (2\pi)/\lambda$ and $v(\theta) = v^I(\theta) + jv^Q(\theta)$ is the signal signature on the $n$-th antenna element [7]. The combined output of the signals of the $N$ elements, $\widetilde{a}\,(k)$, is represented by

$$\widetilde{a}\,(k) = \sum_{n=0}^{N-1} w_n^*\,(k)\,u_n\,(k) = \sum_{n=0}^{N-1} w_n^*(k)u(k)v(\theta). \quad (3)$$

By adjusting the weightings, $w_n\,(k)$, of the arrangement, it is possible to select any direction for the maximum gain [7]. Substituting $u(k)$ of Equation 1 in Equation 3 gives

$$\tilde{a}(k) = \sum_{n=0}^{N-1}\sum_{i=0}^{L-1} w_n^*(k)\rho_i a(k-\tau_i(k))v(\theta_i) + w_n^*(k)r_n(k) \quad (4)$$

where $r_n(k)$ is the noise associated with each antenna element. The noise is given by the expression $r_n(k) = r_n^I(k) + jr_n^Q(k)$, where $r_n^I(k)$ and $r_n^Q(k)$ are random circular variables with Gaussian distribution, average of zero, and variance $\sigma_r^2$. This noise is known as additive white Gaussian noise (AWGN). It can be seen from Equation 4 that each $i$-th path is weighted by its AOA, $\theta_i$, and by its position in the spatial vector. The spatial processor is therefore able to distinguish and eliminate undesirable paths [7].

## III. Blind Adaptive Antenna Array

The adaptive antena array (Figure 2) is a antenna array (as shown in Figure 1), which operates in conjunction with an algorithm in order to adapt its parameters, $\mathbf{w}(n)$, according to the random variation of the impulse response of the communication channel. The adaptation proceeds by optimizing a cost function, $J\left(\mathbf{w}\,(k)\right)$, in which

$$\begin{aligned}\mathbf{w}\,(k) &= \begin{bmatrix} w_0^I\,(k) \\ \vdots \\ w_{N-1}^I\,(k)\end{bmatrix} + j\begin{bmatrix} w_0^Q\,(k) \\ \vdots \\ w_{N-1}^Q\,(k)\end{bmatrix} \\ &= \mathbf{w}^I\,(k) + j\mathbf{w}^Q\,(k)\end{aligned} \quad (5)$$

where $\mathbf{w}^I\,(k)$ and $\mathbf{w}^Q\,(k)$ are real and imaginary parameters of the adaptive antenna array, respectively.

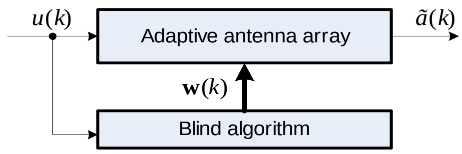



Fig. 2. Structure of a blind adaptive antenna array.

There are two families of adaptive antenna array, either supervised or blind, which differ in the way that the adaptation algorithm operates. In the case of the supervised adaptive antenna array, the algorithm requires a reference, also known as the training sequence, in order to be able to correctly adjust the parameters. With these antenna array, there is some loss of transmission efficiency, since part of the band must be used to transmit this sequence. In contrast, blind adaptive antenna array algorithms do not require a training sequence, and use statistical metrics of the transmitted signal itself in order to adjust the parameters [1]. One of the most well-known blind algorithms is the constant modulus algorithm (CMA) [1].

The CMA attempts to adjust a power integer, $p$, of the information leaving the adaptive filter to a real positive constant, $r_p$. This constant is selected in order to project onto a circle all the points of the output constellation of the adaptive filter [1]. The cost function to be optimized, $J_{CMA}$, is expressed by

$$J_{CMA}\left(\mathbf{w}\,(k)\right) = E\left[e\,(k)^2\right] \quad (6)$$

where $E[\cdot]$ is the mean operator, and

$$e\,(k) = \gamma - \left|\widetilde{a}\,(k)\right|^2, \quad (7)$$

in which $\gamma$ is the dispersion constant given by $\gamma = \frac{E\left\{|a_k|^4\right\}}{E\left\{|a_k|^2\right\}}$, where $a_k$ belongs to the set $M$ of possible modulation symbols employed. The cost function, $J_{CMA}$, is optimized by the gradient descent method with the classical stochastic approximation (substituting the mathematical expectation for an instantaneous estimate). The parameters are adjusted at every instant $k$, according to the expression

$$\mathbf{w}(k) = \mathbf{w}(k-1) + \mu e(k)\,\mathbf{u}(k), \tag{8}$$

where $\mu$ is the adaptation step, and

$$\mathbf{u}(k) = \begin{bmatrix} u(k) & \cdots & u(k-l) & \cdots & u(k-N+1) \end{bmatrix}^T. \tag{9}$$

Nevertheless, as shown previously [1], the Godard criterion, used in the CMA algorithm, possesses points of local minima that can hinder the ISI reduction.

## IV. Blind array of adaptive antennas based on linear programming

### A. Cost function

According to earlier work [3], [4], [5], the convex cost function for the blind adaptive temporal filter can be expressed by

$$\begin{aligned} J_{PL}(\mathbf{w}) &= J_{PL}\left(\mathbf{w}^I, \mathbf{w}^Q\right) \\ &\equiv \max\left|\tilde{a}^I(k)\right| + \max\left|\tilde{a}^Q(k)\right| \end{aligned} \tag{10}$$

and this same scheme can be used for blind adaptive spatial filters, given the similarity in the objectives, which is to reduce the ISI. Hence, expanding the terms $\tilde{a}^I(k)$ and $\tilde{a}^Q(k)$, using Equation 4, gives

$$\begin{aligned} &\max\left|\tilde{a}^I(k)\right| = \\ &\max\left|\sum_{n=0}^{N-1}\sum_{i=1}^{L-1}\rho_i^I(k)a^I\left(k-\tau_i(k)\right)v^I(\theta_i)w_n^I(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}\sum_{i=1}^{L-1}-\rho_i^I(k)a^Q\left(k-\tau_i(k)\right)v^Q(\theta_i)w_n^I(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}\sum_{i=0}^{L-1}-\rho_i^Q(k)a^Q\left(k-\tau_i(k)\right)v^I(\theta_i)w_n^I(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}\sum_{i=0}^{L-1}-\rho_i^Q(k)a^I\left(k-\tau_i(k)\right)v^Q(\theta_i)w_n^I(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}\sum_{i=1}^{L-1}-\rho_i^I(k)a^I\left(k-\tau_i(k)\right)v^Q(\theta_i)w_n^Q(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}\sum_{i=1}^{L-1}-\rho_i^I(k)a^Q\left(k-\tau_i(k)\right)v^I(\theta_i)w_n^Q(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}\sum_{i=1}^{L-1}-\rho_i^Q(k)a^I\left(k-\tau_i(k)\right)v^I(\theta_i)w_n^Q(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}\sum_{i=0}^{L-1}\rho_i^Q(k)a^Q\left(k-\tau_i(k)\right)v^Q(\theta_i)w_n^Q(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}r^I(k)w_n^I(k)\right| \quad +\max\left|\sum_{n=0}^{N-1}-r^Q(k)w_n^Q(k)\right|, \end{aligned} \tag{11}$$

and

$$\begin{aligned} &\max\left|\tilde{a}^Q(k)\right| = \\ &\max\left|\sum_{n=0}^{N-1}\sum_{i=1}^{L-1}\rho_i^I(k)a^I\left(k-\tau_i(k)\right)v^Q(\theta_i)w_n^I(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}\sum_{i=0}^{L-1}\rho_i^Q(k)a^I\left(k-\tau_i(k)\right)v^I(\theta_i)w_n^I(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}\sum_{i=1}^{L-1}\rho_i^I(k)a^Q\left(k-\tau_i(k)\right)v^I(\theta_i)w_n^I(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}\sum_{i=0}^{L-1}-\rho_i^Q(k)a^Q\left(k-\tau_i(k)\right)v^Q(\theta_i)w_n^I(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}\sum_{i=1}^{L-1}\rho_i^I(k)a^I\left(k-\tau_i(k)\right)v^I(\theta_i)w_n^Q(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}\sum_{i=1}^{L-1}-\rho_i^Q(k)a^I\left(k-\tau_i(k)\right)v^Q(\theta_i)w_n^Q(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}\sum_{i=0}^{L-1}-\rho_i^I(k)a^Q\left(k-\tau_i(k)\right)v^Q(\theta_i)w_n^Q(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}\sum_{i=0}^{L-1}\rho_i^Q(k)a^Q\left(k-\tau_i(k)\right)v^I(\theta_i)w_n^Q(k)\right| \\ &+\max\left|\sum_{n=0}^{N-1}r^Q(k)w_n^I(k)\right| + \max\left|\sum_{n=0}^{N-1}r^I(k)w_n^Q(k)\right|. \end{aligned} \tag{12}$$

This assumes that the transmitted signal, $a(k)$, is modulated in the $M$-QAM scheme, where

$$M \equiv \max\left|a^I(k)\right| = \max\left|a^Q(k)\right| \text{ for any } k, \tag{13}$$

and that

$$\max\left|r^I(k)\right| = \max\left|r^Q(k)\right| = \sigma_r \text{ for any } k \tag{14}$$

where $\sigma_r$ is the radius associated with the circular AWGN, $r(k)$. Equation 11 can then be simplified and expressed by

$$\begin{aligned} \max\left|\tilde{a}^I(k)\right| = M\sum_{n=0}^{N-1}\sum_{i=0}^{L-1}&\left(\left|\rho_i^I(k)v^I(\theta_i)w_n^I(k)\right|\right. \\ &+\left|\rho_i^I(k)v^Q(\theta_i)w_n^I(k)\right| + \left|\rho_i^Q(k)v^Q(\theta_i)w_n^I(k)\right| \\ &+\left|\rho_i^Q(k)v^I(\theta_i)w_n^I(k)\right| + \left|\rho_i^I(k)v^I(\theta_i)w_n^Q(k)\right| \\ &+\left|\rho_i^Q(k)v^I(\theta_i)w_n^Q(k)\right| + \left|\rho_i^I(k)v^Q(\theta_i)w_n^Q(k)\right| \\ &\left.+\left|\rho_i^Q(k)v^Q(\theta_i)w_n^Q(k)\right|\right) \\ +\sigma_r\sum_{n=0}^{N-1}&\left|w_n^I(k)\right| + \left|w_n^Q(k)\right|. \end{aligned} \tag{15}$$

Similar to Equations 11 and 15, the term $\tilde{a}^Q(k)$ can be expressed by

$$\max\left|\tilde{a}^Q(k)\right|=M\sum_{n=0}^{N-1}\sum_{i=0}^{L-1}\Big(\left|\rho_i^I(k)v^I(\theta_i)w_n^I(k)\right| + \left|\rho_i^I(k)v^Q(\theta_i)w_n^I(k)\right| + \left|\rho_i^Q(k)v^Q(\theta_i)w_n^I(k)\right| + \left|\rho_i^Q(k)v^I(\theta_i)w_n^I(k)\right| + \left|\rho_i^I(k)v^I(\theta_i)w_n^Q(k)\right| + \left|\rho_i^Q(k)v^I(\theta_i)w_n^Q(k)\right| + \left|\rho_i^I(k)v^Q(\theta_i)w_n^Q(k)\right| + \left|\rho_i^Q(k)v^Q(\theta_i)w_n^Q(k)\right|\Big) + \sigma_r\sum_{n=0}^{N-1}\left|w_n^I(k)\right| + \left|w_n^Q(k)\right|. \quad (16)$$

The functions $\rho_i^I(k)v^I(\theta_i)w_n^I(k)$, $\rho_i^I(k)v^Q(\theta_i)w_n^I(k)$, $\rho_i^Q(k)v^Q(\theta_i)w_n^I(k)$, $\rho_i^Q(k)v^I(\theta_i)w_n^I(k)$, $\rho_i^I(k)v^I(\theta_i)w_n^Q(k)$, $\rho_i^Q(k)v^I(\theta_i)w_n^Q(k)$, $\rho_i^I(k)v^Q(\theta_i)w_n^Q(k)$, and $\rho_i^Q(k)v^Q(\theta_i)w_n^Q(k)$ are linear in $\mathbf{w}^I$ and $\mathbf{w}^Q$, and their moduli $|\rho_i^I(k)v^I(\theta_i)w_n^I(k)|$, $|\rho_i^I(k)v^Q(\theta_i)w_n^I(k)|$, $|\rho_i^Q(k)v^Q(\theta_i)w_n^I(k)|$, $|\rho_i^Q(k)v^I(\theta_i)w_n^I(k)|$, $|\rho_i^I(k)v^I(\theta_i)w_n^Q(k)|$, $|\rho_i^Q(k)v^I(\theta_i)w_n^Q(k)|$, $|\rho_i^I(k)v^Q(\theta_i)w_n^Q(k)|$, and $|\rho_i^Q(k)v^Q(\theta_i)w_n^Q(k)|$ are convex functions in $\mathbf{w}^I$ and $\mathbf{w}^Q$. Hence, as $J_{PL}\left(\mathbf{w}^I,\mathbf{w}^Q\right)$ is a sum of these functions, it can be said that $J_{PL}\left(\mathbf{w}^I,\mathbf{w}^Q\right)$ is also convex. Without any restriction, $J_{PL}\left(\mathbf{w}^I,\mathbf{w}^Q\right)$ possesses a trivial global minimum in $\mathbf{w}^I=\mathbf{w}^Q=0$, generating $\tilde{a}(k)=0$ as output. In order to make $J_{PL}\left(\mathbf{w}^I,\mathbf{w}^Q\right)$ practicable, the following restriction is proposed:

$$\sum_{n=0}^{N-1} v^I(\theta_d)w_n^I(k) - v^Q(\theta_d)w_n^Q(k) = 1, \quad (17)$$

where $\theta_d$ is the AOA of the desired signal. This restriction is based on the initial condition of the CMA algorithm for temporal filters [1], and on the proposals described previously [3], [4], [5]. The use of this restriction can avoid solutions with null outputs, and at the same time ensure the functioning of the antenna array. The linearity of the restriction means that the convex nature of the cost function is maintained, together with the global convergence of the function.

*B. Linear programming*

Based on Equations 10, 15, and 12, the gains of the arrays of adaptive antennas can be adjusted using the following linear programming strategy:

$$\min \begin{cases} \max\left|\tilde{a}^I(k)\right| + \max\left|\tilde{a}^Q(k)\right| \\ \vdots \\ \max\left|\tilde{a}^I(k-K+1)\right| + \max\left|\tilde{a}^Q(k-K+1)\right| \end{cases}$$
$$\text{s.t.} \quad \left\{ \sum_{n=0}^{N-1} v^I(\theta_d)w_n^I(k) - v^Q(\theta_d)w_n^Q(k) = 1, \right. . \quad (18)$$

Substituting the values of $\tilde{a}^I(k)$ and $\tilde{a}^Q(k)$ for Equation 3, and using the same strategy presented previously [3], [4], [6] of introducing two auxiliary variables, $\tau_1$ and $\tau_2$, the linear programming strategy presented in Equation 18 can be expressed minimizing the equation

$$M\tau_1 + M\tau_2, \quad (19)$$

subject to the following constraints:

$$\sum_{n=0}^{N-1} w_n^I(k)u^I(k) - w_n^Q(k)u^Q(k) \le M\tau_1 \quad (20)$$
$$\vdots$$
$$\sum_{n=0}^{N-1} w_n^I(k)u^I(k-K+1) - w_n^Q(k)u^Q(k-K+1) \le M\tau_1, \quad (21)$$

$$\sum_{n=0}^{N-1} w_n^I(k)u^I(k) - w_n^Q(k)u^Q(k) \ge -M\tau_1 \quad (22)$$
$$\vdots$$
$$\sum_{n=0}^{N-1} w_n^I(k)u^I(k-K+1) - w_n^Q(k)u^Q(k-K+1) \ge -M\tau_1, \quad (23)$$

$$\sum_{n=0}^{N-1} w_n^Q(k)u^I(k) - w_n^I(k)u^Q(k) \le M\tau_2 \quad (24)$$
$$\vdots$$
$$\sum_{n=0}^{N-1} w_n^Q(k)u^I(k-K+1) - w_n^I(k)u^Q(k-K+1) \le M\tau_2, \quad (25)$$

$$\sum_{n=0}^{N-1} w_n^Q(k)u^I(k) - w_n^I(k)u^Q(k) \ge -M\tau_2 \quad (26)$$
$$\vdots$$
$$\sum_{n=0}^{N-1} w_n^Q(k)u^I(k-K+1) - w_n^I(k)u^Q(k-K+1) \ge -M\tau_2, \quad (27)$$

and

$$\sum_{n=0}^{N-1} v^I(\theta_d)w_n^I(k) - v^Q(\theta_d)w_n^Q(k) = 1. \quad (28)$$

At the optimum condition, $M\tau_1 = \max\left|\tilde{a}^I(k)\right|$ and $M\tau_2 = \max\left|\tilde{a}^Q(k)\right|$. The linear programming strategy is composed of $2N+2$ variables, with $4K+1$ linear restrictions. In practice, the variable $N$ is associated with the number of antennas in the array (see Section II), and the variable $K$ represents the number of repetitions of the signal that must be stored in order to generate the optimization curve.

*C. Architecture of the BAAA-LP*

The BAAA-LP (with the structure presented in Figure 3) operates in blocks, with a buffer, $K$, storing the vectors, $\mathbf{u}(k)$, and constructing the matrices

$$\mathbf{U}^I(k) = \left[\begin{array}{ccc} \mathbf{u}^I(k) & \cdots & \mathbf{u}^I(k-K+1) \end{array}\right] \quad (29)$$

and

$$\mathbf{U}^Q(k) = \left[\begin{array}{ccc} \mathbf{u}^Q(k) & \cdots & \mathbf{u}^Q(k-K+1) \end{array}\right]. \quad (30)$$

In contrast to the CMA, which updates the parameters every instant $k$, the BAAA-LP updates the parameters for each block $K$ of symbols. For static channels, this does not present any difficulty, but for mobile channels the value of $K$ must be smaller than the coherence time of the channel.

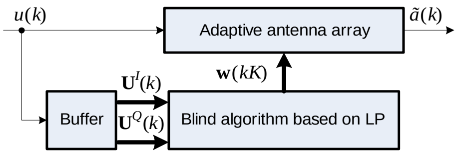



Fig. 3. Structure of a blind linear adaptive antenna array based on LP (BAAA-LP).

## V. Results obtained

In order to validate use of the BAAA-LP and assess its reliability and performance, simulations were performed for the 16-QAM and 64-QAM digital communication systems, without channel encoding and operating at a symbol rate of $1/T_s$. The system was modeled and simulated using the Matlab 2013a platform, according to the scheme shown in Section II, and the results were analyzed using the interior-point, active-set, and simplex methods, implemented using the optimization toolbox of Matlab. In addition to the LP techniques, the results obtained using the CMA algorithm were also analyzed, in order to compare the performance of the BAAA-LP with that of one of the most widely used methods for blind adaptation of antenna arrays. The active-set method implemented using the optimization toolbox of Matlab is a variation of the sequential quadratic programming method, in which the quadratic term is reduced to zero.

The simulations were used to generate curves of the bit error rate (BER) as a function of $E_b/N_0$. The results were analyzed for a scenario of a channel with ISI, described in Table I, with the parameters given in Table II.

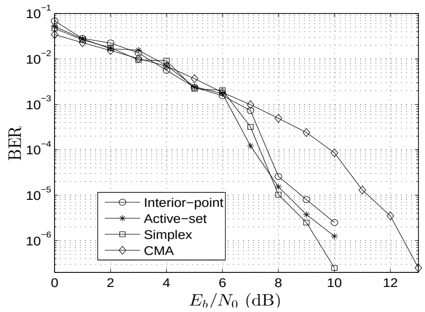
TABLE I
Data describing the simulated communication channel.

| | | | | |
|---|---|---|---|---|
| $\rho_0 = 1$ | $\rho_1 = 0.7$ | $\rho_2 = 0.5$ | $\rho_3 = 0.3$ | $\rho_4 = 0.1$ |
| $\tau_0 = 0$ | $\tau_1 = 5Ts$ | $\tau_2 = 7T_s$ | $\tau_3 = 12Ts$ | $\tau_4 = 15Ts$ |
| $\theta_0 = 60^o$ | $\theta_1 = 0^o$ | $\theta_2 = 30^o$ | $\theta_3 = 90^o$ | $\theta_4 = 120^o$ |

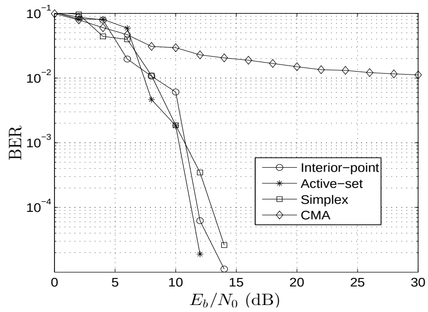
TABLE II
Parameters used in the simulations.

| | |
|---|---|
| Number of antennas ($N$) | 8 |
| Number of repetitions of the signal ($K$) | 200 |
| Adaptation step ($\mu$) para o CMA | 0.01 |
| Number of simulated symbols | $1 \times 10^7$ |

Figures 4(a) and 4(b) show the results obtained for the 16-QAM and 64-QAM systems, respectively. In the case of the 16-QAM system, there was a gain of approximately 2.5 dB, comparing the results of the linear programming methods with the traditional CMA method. For the 64-QAM system, the results showed that the CMA method was unable to achieve convergence to a global minimum, unlike the methods based on linear programming.




(a) 16-QAM system




(b) 64-QAM system

Fig. 4. Performance curve of BER as a function of $E_b/N_0$ for the channel described in Table I.

## VI. Conclusions

This work presents a convex optimization proposal, based on linear programming, for blind adaptive antenna arrays applied to wireless digital communication systems. Different to previous proposals described in the literature, a new model has been developed for channels with AWGN and ISI. A comparative analysis is also presented of different linear programming optimization techniques applied to the proposed model. The proposed structure was submitted to simulations and compared with a conventional system in order to evaluate its performance in different transmission channel models. The significance of the results obtained suggests that the proposed scheme could be used in real wireless communication systems.

## References

[1] D. Godard, "Self-recovering equalization and carrier tracking in two-dimensional data communication systems," *Communications, IEEE Transactions on*, vol. 28, no. 11, pp. 1867–1875, 1980.

[2] D. Li and Y. Wang, "Study of smart antenna beamformer based on constant modulus algorithm," in *Information and Computing (ICIC), 2011 Fourth International Conference on*, April 2011, pp. 178–180.

[3] R. A. Kennedy and Z. Ding, "Blind adaptive equalizers for quadrature amplitude modulated communication systems based on convex cost functions," *Optical Engineering*, vol. 31, no. 6, pp. 1189–1199, 1992.

[4] Z. Ding and Z.-Q. Luo, "A fast linear programming algorithm for blind equalization," *Communications, IEEE Transactions on*, vol. 48, no. 9, pp. 1432–1436, 2000.

[5] M. A. Fernandes, "Linear programming applied to blind signal equalization," *International Journal of Electronics and Communications (AEU)*, vol. 69, no. 1, pp. 408 – 417, 2015.

[6] Z.-Q. Luo, M. Meng, K. M. Wong, and J.-K. Zhang, "A fractionally spaced blind equalizer based on linear programming," *Signal Processing, IEEE Transactions on*, vol. 50, no. 7, pp. 1650–1660, 2002.

[7] Joseph C. Liberti and Theodore S. Rappaport, *Smart Antennas for Wireless Communications: IS-95 and Third Generation CDMA Applications*. Prentice-Hall PTR, 1990.