Investigating the potential of UAV for gunshot DoA estimation and shooter localization

Rigel P. Fernandes[†], Angelo M. R. C. Borzino[†], António L. L. Ramos[‡], and José A. Apolinário Jr.[†]

***Abstract*— This paper presents an approach to obtain stationary shooter localization in a Cartesian plane using a moving array of microphones. A single array is embedded in an unmanned aerial vehicle (UAV), a quadcopter, to explore benefits provided by its intrinsic mobility. We propose a model which is based on geometrical acoustics and that assumes the gunshot signals as being buried in strong noise generated by propellers. Generalized cross correlation algorithms are used to estimate the Direction of Arrival (DoA) of the impulsive gunshot signals and, finally, bearings-only target motion analysis techniques are applied to estimate potential shooters localization from the noisy DoAs.**

***Keywords*— UAV, shooter localization, generalized cross-correlation, TDoA, signal enhancement, LS, TLS.**

I. INTRODUCTION

Shooter localization and Unmanned Aerial Vehicles (UAV) are currently subjects of great interest and have been successfully used in many different applications ranging from defense and peacekeeping forces to surveillance and law-enforcement. Specifically, shooter localization systems have been reported to benefit populations by reducing firearm-related violence [1]. As suggested in [2], these systems can also be used to help protecting endangered animals from poachers. UAV offers great flexibility in search and rescue missions due to their ability to cover large areas quickly and efficiently [3], [4].

Acoustic signals have been used successfully to estimate gunshot direction of arrival (DoA) and shooter localization. The devices currently available are stand alone systems. For instance, the vehicle-mountable Boomerang system and stationary structures composed of a microphone array [5], [6].

Two main acoustic signatures may be produced by a gunshot, namely muzzle blast (MB) and shockwave. The MB is an acoustic wave related to the explosion in the gun muzzle. This wave propagates from the muzzle in all directions at the speed of sound. Thus, estimating the DoA of this component, using a stationary system, produces only information about shooter direction. The shock wave is an acoustic wave created by supersonic bullets that provides information about bullet trajectory and allows to determine the shooter's location, if MB is also available [7]. Another approach to estimate shooter localization when only MB is available is to use an elevated platform to capture gunshot acoustic wave and use a digital map to estimate the intersection point of the estimated angles (azimuth and zenith) and the ground surface [6].

UAV has been associated with military applications due to its non life-threatening operability. Recent researches regarding UAV rely on images captured by built-in cameras to sense the environment. These signals allow researchers to generate a suitable waypoint for navigation or a trajectory for an autonomous formation flight of UAVs [8], [9]. Moreover, quadcopter has been widely used due to its characteristics, such as vertical takeoff, stabilization, and gyroscope and accelerometer signals availability [10].

In [11], Ponda proposes a trajectory optimization to target localization using a small UAV. Bearings-only target motion analysis (BO-TMA) is used to estimate trajectory of a target using bearings. These are estimated with image signals captured by a camera attached in the UAV.

In [4], it was proposed an array of microphones embedded in an UAV quadcopter to capture audio signals to find distress people in disaster situations.

Nevertheless, none of these works that have been put forward present a method to capture gunshot signals using an array of microphones embedded in an quadcopter and use these signals to estimate DoA (or bearing) of a shooter (or target). In addition, in many real situations, like crossfire in urban areas, a set of gunshots is available. This allows the use of BO-TMA techniques to estimate shooter location.

It is known that UAV can be useful to shooter localization, however one should be aware of the main issue that arises when the microphone array is attached to an UAV, the noise caused by propellers [12].

Despite the fact that UAV noise may reduce significantly the range for DoA estimation of acoustic waves, Exhaustive Search (ES) [13] and Iterative Least Squares (ILS) [14] are algorithms that could be used to estimate DoA. These algorithms were created to turn DoA estimation more robust to highly noisy signals.

In the case of a set of gunshot signals captured by UAV in different positions, BO-TMA should be used to estimate shooter location. This would allow to determine shooter localization without resorting to the presence of the shock wave component [7] or a highly accurate digital map [6].

In this paper, new possibilities offered by a moving array of microphones are explored and issues regarding noise caused by UAV propellers are analyzed. It is meant to be a preliminary study to assess through simulations if it is possible to estimate DoA and target localization using gunshot signals buried on

Rigel P. Fernandes, Angelo M. R. C. Borzino, and José A. Apolinário Jr. are with the Program of Defense Engineering, Military Institute of Engineering (IME), Rio de Janeiro-RJ, Brazil, E-mails: `rigelfernandes@gmail.com`, `borzino@ime.eb.br`, `apolin@ime.eb.br`. António L. L. Ramos is with the University College of Southeast Norway (HSN), Kongsberg, Norway, E-mail: `antonio.ramos@hbv.no`. This work was partially supported by Brazilian Agencies CAPES (Project Prodefesa, Process no. 23038.009094/2013-83), CNPq (Project Universal, Process no. 45901/2014-0), FAPERJ (Process no. E-26/203.539/2015), and the Brazilian Navy.

UAV noise, as well as determining the range of effectiveness for DoA estimation. In this paper we focus on DoA estimation and investigate possibilities to use BO-TMA techniques to estimate localization. It also shows results regarding signal enhancement of gunshot noisy signals.

The paper is organized as follows: Section II describes techniques employed to estimate shooter localization, namely generalized cross-correlation (GCC) and BO-TMA algorithms. It also points out signal enhancement techniques that could improve results. Section III explains signal modelling and simulations. Section IV shows and discusses experimental results. Section V concludes the paper.

II. The Shooter Localization Problem

A. Problem statement and assumptions

For the two-dimensional target localization problem, we assume that target (represented by its coordinate vector $\mathbf{p}$) is stationary. Also, we assume exact observer positions $\mathbf{r}_k$ in the exact instant that the gunshot signals are acquired. The frequency that gunshot occurs is assumed constant during the finite observation time interval $1 \leq k \leq N$.

We also assume that:

- Only a set of MB is available in this work to obtain DoA measurement $\hat{\theta}_k$, i.e., the array of microphones is out of the shock wave field of view in the case of supersonic shots.
- The observer position measurements are not subject to noise, i.e., there are no errors in coordinates x and y of the observer positions.
- The target is observable, i.e., it is possible to estimate a unique position with noise free DoAs [15]; for instance, when the UAV is not flying towards the target.

Figure 1 shows a TMA geometry where all these assumptions are observed.

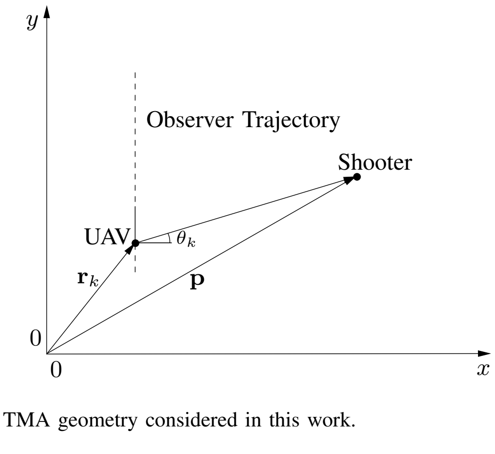

Fig. 1. TMA geometry considered in this work.

B. Signal Enhancement

There are a myriad of methods that can be employed in signal enhancement (SE) applied to gunshot signals. For instance, adaptive filtering, spectral subtraction, and median filtering [16], [17], [18]. As one of the main goals in this paper is to estimate DoA from noisy signals, it is natural to consider signal enhancement as a way to improve estimations. Note that all algorithms before mentioned have shown good results when the signal-to-noise ratio (SNR) is low due to the large distance between the shooter and the array of microphones, i.e., the signals are highly corrupted by observation noise.

Recently, the use of adaptive filtering has shown to be a way to cancel the noise caused by propellers. Fernandes [12] advocates the use of piezoelectric sensors to capture signals correlated only to the noise caused by propellers. This permits the use of these signals as the input signals of the adaptive filter. Fernandes used a low-cost small UAV with only one propeller rotating. It was meant to be a preliminary study to find out possibilities of using this kind of sensor.

However, in the present work, the UAV has four propellers. This fact turns the problem more complex. In addition, there is a large number of embedded systems that produce strong electromagnetic interference. Also, it is likely that the signals captured by the sensors present some non-linearities. All these cause the signal captured by piezoelectric sensor not only to have a weaker correlation with the noise captured by a microphone but also to be prone to present harmonics not present in the latter.

In addition, we figured out that the Magnitude Squared Coherence (MSC) using Welch's averaged modified periodogram [19] in [12], presented a minimum score equal to 0.60 (in a scale from 0, uncorrelated, to 1, fully correlated) while the signals captured by the actual UAV have a score of 0.25 (in average). This signal enhancement method will not be used in this work.

Borzino [18] proposed the use of median filtering (MF) to enhance gunshot signals. Median filtering has been used to enhance harmonic parts of music by removing undesired impulsive signals. This filter can be easily modified to enhance impulsive signals by subtracting harmonic part (output of median filtering) from the original signal; as a result, only the muzzle blast component would remain after the filtering process.

Spectral subtraction is another approach to signal enhancement. This technique needs information about the noise to estimate its spectrum. Subtracting spectrum of the noise from the original signal yields the enhanced signal [17].

C. DoA estimation

Many methods have been proposed to tackle the DoA estimation problem [20]. Among these algorithms, GCC can be used to gunshot signals, a broadband signal in nature. The GCC method uses time difference of arrival (TDoA) between signals that reach pairs of sensors to estimate DoA. TDoA can be estimated from the peak of the cross-correlation between the signals. The GCC-PHAT (Phase Transform) is a subclass of the GCC method that is used in this work.

Different techniques aim at maximizing the effectiveness of the GCC-PHAT, including algorithms ILS [14] and ES [13]. They select a given number of pairs of microphones based on a least-squares cost function, in an attempt to minimize errors due to spurious signals. The ILS algorithm eliminates the pairs of microphones that less contribute to the minimization of the cost function until we have six or five pairs [14]. The ES(n) algorithm evaluates the cost function for all possible combinations of n pairs and selects the combination that produces the lowest cost function value.

D. Shooter Localization

Many TMA methods that use different measures can be considered to localize a target. For instance, bearings-only (BO-TMA) [21], Doppler bearings (DB-TMA) [22], range-only (RO-TMA) [23], and power bearings (PB-TMA) [24]. In this work the only information available about the shooter is the estimated DoAs (or bearings) which in turn are estimated from the MB. Thus, a method to estimate shooter localization using only MB DoAs would be suitable.

The fact that it is not always that a stationary array of microphones detects both MB and shock wave acoustic signatures to estimate target localization [25] reinforces the need for a method able to estimate shooter localization based on MB DoAs only.

As a result, bearings-only technique is considered in this work to the shooter localization problem. Passive BO-TMA has been studied since the sixties [26]. This problem arises in many situations, e.g. acoustic (passive sonar), electromagnetic (aircraft surveillance), optic (optronic sensors). Classical BO-TMA algorithms try to estimate 4 parameters (two coordinates for geographical position, velocity, and course of a target). In this work we are interested in the first two parameters, coordinates x and y [26].

A considerable number of bearings-only algorithms have been proposed. In this work, we consider the Linear Least-Squares (LS) and the Total Least Squares (TLS) for shooter position estimation from the corrupted acoustic data [21].

Doğançay [21] stated that the Linear least-squares can be writen as

$$\hat{\mathbf{p}}_{\mathbf{LS}} = (\mathbf{A}^{\mathrm{T}}\mathbf{W}^{-1}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{W}^{-1}\mathbf{b}, \quad (1)$$

where $N \times 2$ matrix $\mathbf{A}$ is defined as

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_N \end{bmatrix}^{\mathrm{T}}, \quad (2)$$

with vectors $\mathbf{a}_k$ being given by

$$\mathbf{a}_k = \begin{bmatrix} \sin\hat{\theta}_k & -\cos\hat{\theta}_k \end{bmatrix}^{\mathrm{T}}, \quad (3)$$

while the $N \times 1$ vector $\mathbf{b}$ and the $N \times N$ diagonal weighting matrix $\mathbf{W}$ are defiend, respectively, as

$$\mathbf{b} = \begin{bmatrix} \mathbf{a}_1^{\mathrm{T}}\mathbf{r}_1 & \mathbf{a}_2^{\mathrm{T}}\mathbf{r}_2 & \cdots & \mathbf{a}_N^{\mathrm{T}}\mathbf{r}_N \end{bmatrix}^{\mathrm{T}}, \quad (4)$$

and

$$\mathbf{W} = \begin{bmatrix} d_1^2\sigma_{n_1}^2 & & & \mathbf{0} \\ & d_2^2\sigma_{n_2}^2 & & \\ & & \ddots & \\ \mathbf{0} & & & d_N^2\sigma_{n_N}^2 \end{bmatrix}. \quad (5)$$

Also, according to Doğançay [21], the total least squares solution can be writen as

$$\hat{\mathbf{p}}_{\mathbf{TLS}} = \frac{1}{v33}\begin{bmatrix} v13 \\ v23 \end{bmatrix} \quad (6)$$

where v_{13}, v_{23}, v_{33} are the elements in the third column of $\mathbf{V}$, the unitary matrix obtained from the singular value decomposition of the augmented N × 3 matrix $[\mathbf{A}, -\mathbf{b}]$.

The total least squares solution is, in general, expected to exhibit smaller estimation bias than the LS solution [21].

III. MODELING AND MEASUREMENT

Figure 1 shows the TMA geometry considered in this work with a single observer and a single stationary target. For each gunshot, the signal that would arrive at the array at instant t_k is modeled by delayed copies of a real gunshot signal according to the array geometry and angle θ_k.

The array coordinates were set according to a possible implementation on a quadcopter Parrot AR Drone 2.0. Thus, the simulated array is composed of five microphones. One microphone attached to the base of the hull and the four remaining microphones at the cross of UAV, near the propellers. Each microphone near propeller would be protected from the wind caused by propellers.

To simulate signals captured with this array, we corrupted the delayed gunshot signals with real quadcopter noise. The UAV noise samples were acquired using a omnidirectional microphone placed at the hull of UAV when the UAV was hovering in a silent environment. This noise is added with the gunshot signals with an appropriate SNR.

In order to estimate the SNR, measurements were made for distances 41, 117, 149, 174, and 200 meters. All gunshot signals used in this estimation problem were originated by a 7.62mm rifle and recorded by a stationary array (without quadcopter noise). Expressing, for a distance d, $x_1[n] = s[n] + n_1[n]$, we were able to estimate σ_s^2 and $\sigma_{n_1}^2$ as $\hat{\sigma}_s^2 = \hat{\sigma}_{x_1}^2 - \hat{\sigma}_{n_1}^2$ (from a 7.5ms window containing the muzzle blast component), $\hat{\sigma}_{n_1}^2$ was obtained from a 7.5ms window with environmental noise only.

This initial estimation does not consider the noise caused by UAV propellers. In this work, the SNR is key at simulating gunshot signals and we must consider the UAV noise for more realistic simulations. As the distance between propellers and microphone array does not change and assuming constant speed of propellers (when UAV is hovering), the variance of the additive noise $n_2[n]$, $\sigma_{n_2}^2$, may be considered constant. From data acquisition of the UAV in a quite environment, we were able to obtain $\hat{\sigma}_{n_2}^2$, an estimate of the variance $\sigma_{n_2}^2$ of the propellers noise $n_2[n]$.

Assuming that recordings (gunshots and UAV noise) were carried in similar conditions (same pre-amp gain, same microphones, etc.) we were able to compute the values of the SNR for a simulated UAV recorded gunshot signal, which is given by

$$SNR = 10log\left(\frac{\hat{\sigma}_s^2}{\hat{\sigma}_{n_1}^2 + \hat{\sigma}_{n_2}^2}\right) \quad (7)$$

and are shown in Figure 2. In this Figure, an interpolated curve of SNR (in dB) is shown for a range of distances from 20 to 200m. In order to verify the exactness of the estimations of Figure 2 three gunshot signals were recorded with a microphone embedded in a quadcopter (Parrot AR 2.0) while it was hovering. The distance between the quadcopter and shooter is 102m. The estimated SNR of these signals is 3.1 dB, which is a higher value than the presented in Figure 2 (1.5 dB for $d = 102$ meters).

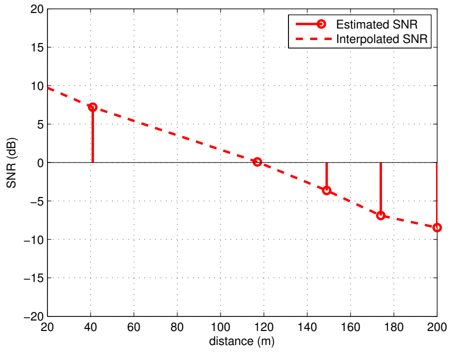

Fig. 2. Estimated SNR×distance for a 7.62mm assault rifle (FAL).

IV. Results and Discussion

The proposed approach to estimate shooter localization was first evaluated in terms of DoA estimation. Then the BO-TMA algorithms were compared in terms of bias error.

In Table I, the performance of algorithms ES and ILS were compared in terms of angular error. The distance is fixed and the incident angle of the array is random. The ensemble has 1,000 runs. Signal enhancement techniques were combined to the DoA estimation algorithms to evaluate if it is possible to enhance gunshot signals buried in UAV noise. The metric used was the mean squared error (MSE), given by

$$\mathrm{MSE} = \frac{1}{N}\sum_{k=1}^{N}(\theta_k - \hat{\theta}_k)^2. \tag{8}$$

TABLE I

Azimuth MSE for different signal enhancement (SE) and different DoA estimation techniques

Technique		Distance (in m)			
DoA	SE	50	150	190	200
ES(6)	none	0.036	2.124	**6.042**	**7.513**
	MF	**0.005**	**0.877**	35.911	73.465
	SS	0.522	3.648	70.842	161.132
ILS	none	0.423	41.749	**61.683**	**142.497**
	MF	**0.408**	**26.123**	216.462	426.335
	SS	13.248	32.414	240.101	533.242

According to Table I, the smallest MSE in distances of 50 and 150m were achieved using ES(6) (using six pairs from the available $C_2^5 = 10$ pairs of microphones) and median filtering. Waveforms can be viewed in Figure 3. Note that spectral subtraction enhances MB waveform, but additional peaks were created. These peaks may degrade DoA estimation. In fact, the combination of ES and SS resulted in a degraded performance of DoA estimation even for distances as small as 50m. On the other hand, median filtering enhances MB without creating additional peaks.

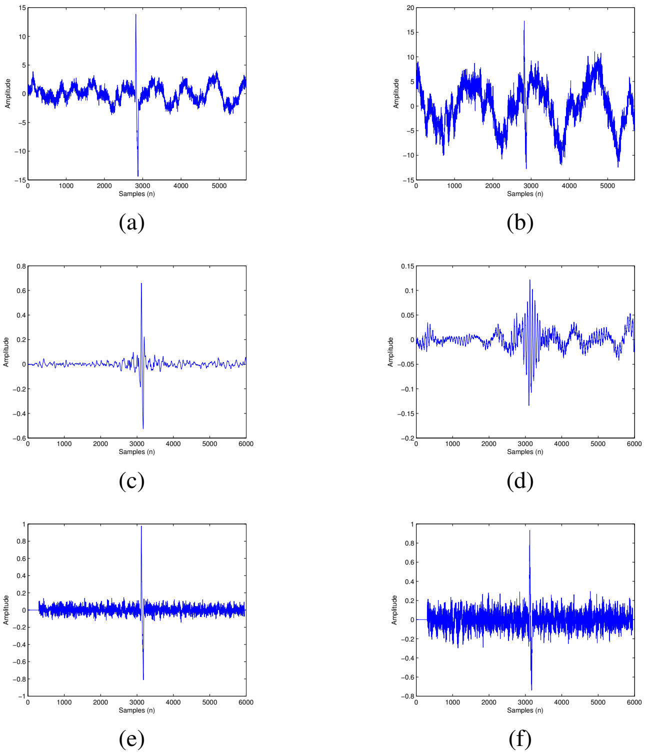

Fig. 3. MB modelled from FAL, 50 meters away from the observer position (left) and 150 meters away from the observer position (right). (a) and (b) without signal enhancement technique. (c) and (d) spectral subtraction. (e) and (f) median filtering.

For distances of 190 and 200 meters, ES has shown to be a robust algorithm, once the error did not increase so much if compared with other combinations of signal enhancement and DoA estimation techniques. In real situations, UAV can be near or far from a shooter when the gunshot occurs. As a result, ES will be used for its robustness.

Table II shows the mean error (in meters) of the distance between true shooter location and the estimated positions, $||\mathbf{p} - \hat{\mathbf{p}}||$. This experiment was carried out considering ten DoAs estimated from ES(6). For BO-TMA, LS and TLS were used; TLS has shown to be the algorithm that produces less bias, except for the distance of 50m where the error is the same. The ensemble had 1,000 independent runs. The distance considered in our results correspond to an average of distances between the first position to the last position of measurements, as depicted in 1.

Table III shows the error when the distance is fixed and the number of measurements varies from 3 to 10. As the number of measurements decreases the error increases.

V. Conclusions

When a set of MB is captured by an UAV in different positions, as usually occurs in a crossfire scenario, a chance to use an statistical approach to estimate shooter localization arises. Herein, we stated that a combination of Exhaustive Search and Total Least Squares can yield good estimations for shooter localization. The accuracy depends on the distance and the number of measurements available. As the distance

TABLE II

LOCALIZATION ERROR $||\mathbf{p}-\hat{\mathbf{p}}||$ AVERAGED WITH TEN MEASUREMENTS) VERSUS DISTANCE

Technique	Distance to shooter (in m)						
BO-TMA	50	100	150	180	190	200	220
LS	**0.22**	1.07	5.86	12.50	15.76	18.76	29.76
TLS	**0.22**	**1.05**	**5.60**	**12.08**	**14.46**	**16.41**	**24.91**

TABLE III

LOCALIZATION ERROR (MEAN DISTANCE 190 METERS)

Technique	# Measurements							
BO-TMA	3	4	5	6	7	8	9	10
LS	103.76	**55.39**	**40.68**	**31.68**	25.51	21.42	18.33	15.76
TLS	**92.76**	64.75	46.51	32.12	**24.44**	**19.91**	**16.95**	**14.46**

increases, the number of measurements used must be larger to maintain the same mean error. Better results could be obtained if the shooter is within the range of 150m, where it is possible, after a first estimation, to refine the result by using median filtering to attain a better estimation. Finally, we conclude that shooter localization using acoustics in an UAV has at least three points to optimize results, namely signal enhancement, DoA estimation, and target localization. The next step is to record real gunshots signals using an array embedded in a quadcopter according to the geometry presented in this paper to see if the results obtained herein prove to be correct.

REFERENCES

[1] J. R. Aguilar, “Gunshot detection systems in civilian law enforcement,” *Journal of the Audio Engineering Society*, vol. 63, no. 4, pp. 280–291, 2015.

[2] M. Hrabina and M. Sigmund, “Acoustical detection of gunshots,” in *Radioelektronika (RADIOELEKTRONIKA), 2015 25th International Conference*, April 2015, pp. 150–153.

[3] P. Doherty and P. Rudol, “A UAV search and rescue scenario with human body detection and geolocalization,” in *AI 2007: Advances in Artificial Intelligence*. Springer, 2007, pp. 1–13.

[4] K. Okutani, T. Yoshida, K. Nakamura, and K. Nakadai, “Outdoor auditory scene analysis using a moving microphone array embedded in a quadrocopter,” in *Intelligent Robots and Systems (IROS), 2012 IEEE/RSJ International Conference on*. IEEE, 2012, pp. 3288–3293.

[5] J. A. Mazurek, J. E. Barger, M. Brinn, R. J. Mullen, D. Price, S. E. Ritter, and D. Schmitt, “Boomerang mobile counter shooter detection system,” in *Defense and Security*. International Society for Optics and Photonics, 2005, pp. 264–282.

[6] D. M. Paredes Calderon and J. A. Apolinário Jr., “Shooter localization based on DoA estimation of gunshot signals and digital map information,” *Latin America Transactions, IEEE (Revista IEEE America Latina)*, vol. 13, no. 2, pp. 441–447, 2015.

[7] I. L. Freire and J. A. Apolinário Jr, “Localização de atirador por arranjo de microfones,” in *Simpósio Brasileiro de Automação Inteligente (in portuguese). SBAI*, vol. X, September 2011, pp. 1049–1053.

[8] H. Alvarez, L. Paz, J. Sturm, and D. Cremers, “Collision avoidance for quadrotors with a monocular camera,” in *Experimental Robotics*. Springer, 2016, pp. 195–209.

[9] S. Mao, W. K. Tan, and K. H. Low, “Autonomous formation flight of indoor uavs based on model predictive control,” in *AIAA Infotech@ Aerospace*, 2016, p. 0515.

[10] K. W. Schram, “Vision-based control and autonomous landing of a VTOL-UAV,” Ph.D. dissertation, Lakehead University, 2014.

[11] S. S. Ponda, R. M. Kolacinski, and E. Frazzoli, “Trajectory optimization for target localization using small unmanned aerial vehicles,” in *AIAA Guidance, Navigation, and Control Conference, Chicago, Illinois, USA*, 2009, pp. 10–13.

[12] R. P. Fernandes, E. C. Santos, A. L. L. Ramos, and J. A. Apolinário Jr., “A first approach to signal enhancement for quadcopters using piezoelectric sensors,” in *Proceedings of The 20th International Conference on Transformative Science and Engineering, Business and Social Innovation*. Society For Design And Process Science. SDPS, 2015, pp. 536–541.

[13] A. M. C. R. Borzino, J. A. Apolinário Jr., and M. L. R. de Campos, “Robust DOA estimation of heavily noisy gunshot signals,” in *Acoustics, Speech and Signal Processing (ICASSP), 2015 IEEE International Conference on*. IEEE, 2015, pp. 449–453.

[14] I. L. Freire, “Robust direction-of-arrival by matched-lags, applied to gunshots,” *The Journal of the Acoustical Society of America*, vol. 135, no. 6, pp. EL246–EL251, 2014.

[15] S. C. Nardone and V. J. Aidala, “Observability criteria for bearings-only target motion analysis,” *Aerospace and Electronic Systems, IEEE Transactions on*, no. 2, pp. 162–166, 1981.

[16] A. L. Ramos, S. Holm, S. Gudvangen, and R. Otterlei, “Real-time vehicle noise cancellation techniques for gunshot acoustics,” in *SPIE Defense, Security, and Sensing*. International Society for Optics and Photonics, 2012, pp. 835 917–835 917.

[17] A. M. C. R. Borzino, J. A. Apolinário Jr., and M. L. R. de Campos, “Estimating direction of arrival of long range gunshot signals,” in *Telecommunications Symposium (ITS), 2014 International*. IEEE, 2014, pp. 1–5.

[18] A. M. C. R. Borzino, J. A. Apolinário Jr., M. L. R. de Campos, and L. W. P. Biscainho, “Signal enhancement for gunshot DOA estimation with median filters,” in *Circuits & Systems (LASCAS), 2015 IEEE 6th Latin American Symposium on*. IEEE, 2015, pp. 1–4.

[19] P. D. Welch, “The use of Fast Fourier Transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms,” *IEEE Transactions on audio and electroacoustics*, vol. 15, no. 2, pp. 70–73, 1967.

[20] H. L. Van Trees, *Optimum array processing - part iv - detection, estimation, and modulation theory, optimum array processing*. John Wiley & Sons, 2004.

[21] K. Doğançay, “Bearings-only target localization using total least squares,” *Signal Processing*, vol. 85, no. 9, pp. 1695–1710, 2005.

[22] K. Ho and Y.-T. Chan, “An asymptotically unbiased estimator for bearings-only and doppler-bearing target motion analysis,” *Signal Processing, IEEE Transactions on*, vol. 54, no. 3, pp. 809–822, 2006.

[23] B. Ristic, S. Arulampalam, and J. McCarthy, “Target motion analysis using range-only measurements: algorithms, performance and application to ISAR data,” *Signal Processing*, vol. 82, no. 2, pp. 273–296, 2002.

[24] G. Gu, “A novel power-bearing approach and asymptotically optimum estimator for target motion analysis,” *Signal Processing, IEEE Transactions on*, vol. 59, no. 3, pp. 912–922, 2011.

[25] R. C. Maher, “Modeling and signal processing of acoustic gunshot recordings,” in *Digital Signal Processing Workshop, 12th-Signal Processing Education Workshop, 4th*. IEEE, 2006, pp. 257–261.

[26] C. Jauffret, D. Pillon, and A.-C. Pignol, “Bearings-only TMA without observer maneuver,” in *Information Fusion, 2008 11th International Conference on*. IEEE, 2008, pp. 1–8.