Performance Analysis of Spectrum Sensing Techniques in Nakagami and Rice Fading Channels

Ricardo A. da S. Júnior, Rausley A. A. de Souza and Dayan A. Guimarães

Abstract— **This paper aims at investigating the performance of four eigenvalue-based techniques for centralized data-fusion cooperative spectrum sensing in cognitive radio networks over flat Nakagami-*m* and Rice fading channels. The detection techniques are the generalized likelihood ratio test (GLRT), the maximum-minimum eigenvalue detection (MMED), the maximum eigenvalue detection (MED), and the energy detection (ED). In the case of Nakagami-*m*, arbitrary fading and phase parameters were assumed, and so was with the Rice parameter in the case of the Rician model.**

Index Terms— cognitive radio, cooperative eigenvalue spectrum sensing, Nakagami, Rice.

I. INTRODUCTION

Nowadays, there is a growing demand for effective use of spectrum and spectral efficient management strategies in the context of fast developing wireless communications systems. The spectrum resources have become scarce, and at the same time there is an increasing demand for better quality of service, as well as higher transmission rates. Nevertheless, in fact there is an artificial scarcity of spectrum, since there are bands that are not actually used during all time in a given region [1]. The cognitive radio (CR) concept [2] can be applied to this context, aims at using the electromagnetic spectrum more efficiently. A CR system uses advanced techniques that optimize the occupation of the bands, and spectrum sensing techniques to find the so-called spectral opportunities within bands of interest in a given area and in a given time. Thus, a CR system makes it possible to use the available spectrum in temporal, spatial and frequency dimensions, without causing interference to licensed systems. Nonetheless, the scenarios of spectral occupancy differ depending of several factors, such as channel conditions, location and prevailing political control of spectrum usage. This implies greater system complexity, since the cognitive cycle of the CR concept includes a step for learning the channel [2]-[3]. Hence, the behavior of the channel, or more precisely the channel model, influences the operation and performance of a CR. Then, evaluating the performance of a CR system under different channel models is of paramount importance. Moreover, the choice of the spectrum sensing technique will also influence the detection performance, depending on the cognitive network architecture and the conditions of the channel. Many detection techniques for spectrum sensing have been proposed so far, e.g. the matched filter, the cyclostationary and the energy detection [4]-[5]. Among the latest ones are those based on the eigenvalues of the received signal covariance matrix; see [6]-[8] and references therein. These techniques have received a lot of attention mainly because they do not require prior information on the transmitted signal, and, in contrast to the energy detection, some eigenvalue-based schemes do not need to know the noise variance either.

No matter the sensing technique adopted, the detection performance depends on the reception conditions of the CRs, and therefore on the propagation environment. For example, in [9] comparisons were made among different models for the energy detector under conditions of additive white Gaussian noise (AWGN) and Rayleigh fading channels. It has been shown that the problem of energy detection lies in the uncertainty of estimating the noise power, which degrades the detection performance [10]-[12]. In [13] the authors analyze the probability of miss detection of the energy detector under Nakagami fading channels. Recently, in [14] the authors presented a new implementation-oriented model in which typical signal processing tasks of a direct-conversion CR receiver were taken into account considering the Rayleigh fading channel.

The aim of this paper is to present the analysis of the spectrum sensing performance under two important channel models: Nakagami-*m* [15] (with arbitrary phase and fading parameters) and Rice [16] (with arbitrary Rice parameter). The Nakagami distribution can be parameterized to model various fading conditions such as Rayleigh and Rice. This means that it is possible to control the severity of the Nakagami fading by making this distribution to fit more appropriately into real scenarios with multipath propagation [13]. The Nakagami-*m* and Rice distributions, which are general, flexible, and easily tractable mathematically, have also been proved useful in practice [17]-[18].

In what concerns the detection technique, we consider the eigenvalue-based generalized likelihood ratio test (GLRT); the maximum-minimum eigenvalue detection (MMED), also known as the eigenvalue ratio detection (ERD); the maximum eigenvalue detection (MED), also known as Roy's largest root test (RLRT); and the energy detection (ED), applied to a centralized data-fusion cooperative spectrum sensing scheme. ED is not an exclusively eigenvalue-based detection technique, but it can be implemented using eigenvalue information. It has been included in the present investigation for the sake of completeness, also giving support to a broader pool of comparisons.

The remainder of this paper is structured as follows. Section II presents the system model for the eigenvalue-based sensing technique and the fading channels models. Section III reports simulation results and discussions concerning the influence of

Ricardo A. da S. Júnior, Rausley A. A. de Souza and Dayan A. Guimarães. National Institute of Telecommunications (Inatel), PO Box 05, 37540-000 Santa Rita do Sapucaí - MG - Brazil, E-mails: ricardojunior@inatel.br, rausley@inatel.br, dayan@inatel.br. This work was partially supported by Fapemig (TEC - APQ-01255-12).

system parameters on the performance of the spectrum sensing. Finally, Section IV concludes the paper.

II. SYSTEM MODEL

A. Centralized Cooperative Eigenvalue Spectrum Sensing

Cooperative spectrum sensing is considered a possible solution for problems experienced by CR networks in a non-cooperative situation, like receiver uncertainty, multipath fading, hidden terminals and correlated shadowing [3].

Consider the well-known baseband memoryless linear discrete-time MIMO fading channel model. Assume that there are ℓ single-antenna CRs, each one collecting n samples of the received signal from k primary transmitters during the sensing period, and that these samples are arranged in a matrix $\mathbf{Y} \in \mathbb{C}^{\ell \times n}$. Similarly, consider that the signal samples from the k primary transmitters are arranged in a matrix $\mathbf{X} \in \mathbb{C}^{k \times n}$, and that $\mathbf{H} \in \mathbb{C}^{\ell \times k}$ is the channel matrix with elements $\{h_{ij}\}$, $i = 1, 2, \ldots, \ell$ and $j = 1, 2, \ldots, k$, representing the channel gain between the j-th primary transmitter and the i-th CR receiver. The elements of the channel matrix $\mathbf{H}$ simulate a flat Nakagami-m or Rice fading channel between each primary transmitter and CR, assumed to be constant during a sensing period and independent from one period to another. Finally, if $\mathbf{V} \in \mathbb{C}^{\ell \times n}$ represents the matrix containing thermal noise samples that corrupt the received signal, then the matrix of collected samples is given by

$$\mathbf{Y} = \mathbf{HX} + \mathbf{V}. \tag{1}$$

In eigenvalue-based spectrum sensing, spectral holes are detected by using test statistics based on the eigenvalues of the received signal sample covariance matrix. In a centralized cooperative scheme with data-fusion, matrix $\mathbf{Y}$ is formed at the fusion center (FC), and the sample covariance matrix

$$\mathbf{R} \cong \frac{1}{n}\mathbf{Y}\mathbf{Y}^{\dagger} \tag{2}$$

is estimated, where † stands for complex conjugate and transpose. The eigenvalues $\{\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_m\}$ of $\mathbf{R}$ are then computed, and assuming a single primary transmitter ($p = 1$), the decision variables for the GLRT, MMED, MED, and ED are respectively calculated according to [7]:

$$T_{\text{GLRT}} = \frac{\lambda_1}{\frac{1}{m}\operatorname{tr}(\mathbf{R})} = \frac{\lambda_1}{\frac{1}{m}\sum_{i=1}^{m}\lambda_i}, \tag{3}$$

$$T_{\text{MMED}} = \frac{\lambda_1}{\lambda_m}, \tag{4}$$

$$T_{\text{MED}} = \frac{\lambda_1}{\sigma^2}, \tag{5}$$

$$T_{\text{ED}} = \frac{\|\mathbf{Y}\|_F^2}{m n \sigma^2} = \frac{1}{m\sigma^2}\sum_{i=1}^{m}\lambda_i, \tag{6}$$

where σ^2 is the thermal noise power, which is assumed to be known and the same in each sensor input, and $\operatorname{tr}(\cdot)$ and $\|\cdot\|_F$ are the trace and the Frobenius norm of the underlying matrix, respectively. The decision upon the occupation of the sensed channel is attained by comparing the test statistics with a decision threshold.

B. The Spectrum Sensing as a Binary Hypothesis Test

Spectrum sensing can be formulated as a binary hypothesis test problem that can be stated as

$$\begin{aligned} &\mathcal{H}_0: \text{ Primary signal is absent} \\ &\mathcal{H}_1: \text{ Primary signal is present,} \end{aligned} \tag{7}$$

where $\mathcal{H}_0$ is the null hypothesis, meaning that there is no licensed user signal active in a specific sensed band, and $\mathcal{H}_1$ is the alternative hypothesis, which indicates that there is at least one active primary user signal.

Two important parameters associated with the assessment of the spectrum sensing performance are the probability of detection, P_{D}, and the probability of false alarm, P_{FA}, which are defined as follows:

$$\begin{aligned} P_{\text{D}} &= \Pr\{\text{decision} = \mathcal{H}_1 \mid \mathcal{H}_1\} = \Pr\{T > \gamma \mid \mathcal{H}_1\} \\ P_{\text{FA}} &= \Pr\{\text{decision} = \mathcal{H}_1 \mid \mathcal{H}_0\} = \Pr\{T > \gamma \mid \mathcal{H}_0\}, \end{aligned} \tag{8}$$

where $\Pr\{\cdot\}$ is the probability of a given event, T the decision variable and γ the decision threshold. The value of γ is chosen depending on the requirements for the spectrum sensing performance, which are typically evaluated through receiver operating characteristic (ROC) curves that show P_{D} versus P_{FA} as they vary with the decision threshold γ. A higher threshold keeps P_{FA} at low levels, but renders detection difficult. On the other hand, a low threshold favors detection, but increases P_{FA}. This tradeoff is clearly seen from the ROC curve.

C. Fading Channels

In [19], the Nakagami complex signal model is discussed considering the statistics of the phase distribution of the channel, besides the envelope distribution, which continues to be a debatable topic. Such distributions, including others as Rayleigh, typically model the envelope of the received signal assuming that the phase distribution is uniform, which is not generally true in real channels. The knowledge of the statistics applied to the phase distribution are important to communication systems analysis, for example when deriving the error probability of digital modulation schemes over fading channels, or when designing or analyzing the performance of carrier-tracking loops. Likewise, the phase distribution is important for analyzing the performance of spectrum sensing schemes, since it will affect the channel between primary transmitters and secondary receivers.

The characterization and modeling of Nakagami channels is still a topic of ongoing research, because the Nakagami process models numerous classes and fading channel conditions, resulting in a model that accurately fits empirical data. Moreover, the Nakagami-m distribution is mathematically simple, facilitating mathematical derivations, and thus making it more attractive for performance analysis. Besides, it brings flexibility and control in the severity of fading, taking for example the case of the Rayleigh fading as a particular situation.

In the model presented in [19], the Nakagami-m phase parameter p is the condition of balance or unbalance between the in-phase and quadrature components of the fading process, which corresponds to the balance or unbalance between the real and imaginary components of the complex Nakagami-m random variate. Thus, in a more general scenario, one can think of an unbalanced structure between components, but still having a total of $2m$ Gaussian processes. The value of p can be in the range $-1 \leq p \leq 1$, with the condition that $p = 0$ leads to the balancing of the generation model, i.e. the same number of real and imaginary Gaussian variates, and imbalanced otherwise. As shown in [19], this condition enables the correct distribution of the envelope and phase of the Nakagami fading process. This means that it is possible to evaluate the performance of the spectrum sensing more appropriately by using this Nakagami-m fading model. Based on the model in [19], the real and imaginary samples of the complex Nakagami-m random variate given by the densities

$$f_X(x) = \left(\frac{m}{\Omega}\right)^{\frac{1+p}{2}m} \frac{|x|^{(1+p)m-1}}{\Gamma\left(\frac{1+p}{2}m\right)} \exp\left(-\frac{m x^2}{\Omega}\right) \qquad (9)$$
$$-\infty < x < \infty$$

$$f_Y(y) = \left(\frac{m}{\Omega}\right)^{\frac{1-p}{2}m} \frac{|y|^{(1-p)m-1}}{\Gamma\left(\frac{1-p}{2}m\right)} \exp\left(-\frac{m y^2}{\Omega}\right) \qquad (10)$$
$$-\infty < y < \infty,$$

were generated by using the inverse cumulative distribution function method [20].

In the Rice fading channel model, fading severity is governed by the Rice parameter K, which is, by definition, the ratio between the powers of the dominant received signal component (referred to as A^2) and those produced by the multipath propagation (referred to as $2\sigma^2$). The higher this ratio, the less severe are the effects of the fading because of the presence of a line-of-sight (LOS) or a dominant multipath component signal. For example, the channel in a cognitive radio system having a LOS with a primary transmitter can be modeled with multiple cases of the Rice fading. So, it is in order to assess the performance of the spectrum sensing process in Rice fading channels. More specifically, it would be interesting to see how the detection performance is impacted by the variation of the Rice parameter. The complex Rice variate was obtained as described in [21].

III. SIMULATION RESULTS

This section presents simulation results and discussions concerning the influence of the Nakagami-m and Rice fading parameters on the spectrum sensing performance.

Figure 1 shows some ROC curves for MED and ED techniques over a Nakagami fading channel, considering the fading parameters $m = 2$ and $m = 7$ and signal-to-noise ratios SNR = –3 and SNR = –6 dB, and $n = 50$ samples per CR. Since the influence of increasing the number ℓ of CRs under cooperation is, as expected, a performance improvement considering fixed the remaining system parameters, we only consider $\ell = 3$. In this scenario we adopt the Nakagami phase parameter as $p = 0$, leading to the balancing of the generation model. From Figure 1 it can be seen that the MED technique is better than ED. Notice that as the fading parameter increases, the sensing performance is improved. This is an expected result, since a larger fading parameter implies a less severe fading. As also expected, the influence of increasing the SNR is a performance improvement, considering fixed the remaining parameters.

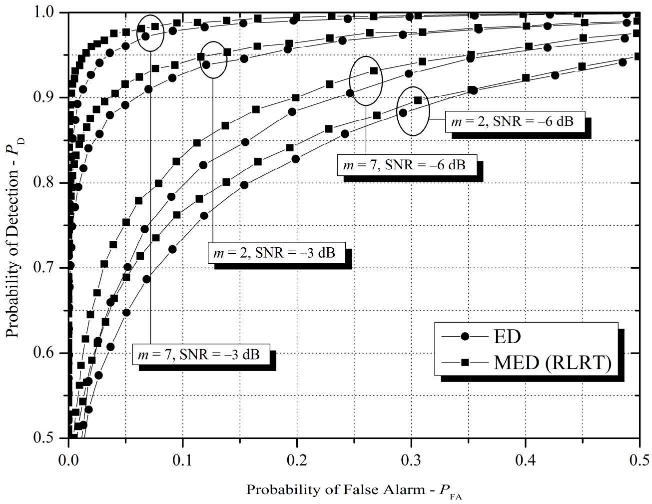

Fig. 1. ROC curves for MED and ED in a Nakagami fading channel under different fading parameter and SNR.

Figure 2 shows ROC curves for the MMED and the GLRT on a Nakagami fading channel, considering the same set of parameters used to plot Figure 1. Here again, as the fading parameter and the SNR increase, the sensing performance is improved. Notice that the GLRT outperforms the MMED in the Nakagami multipath fading channel.

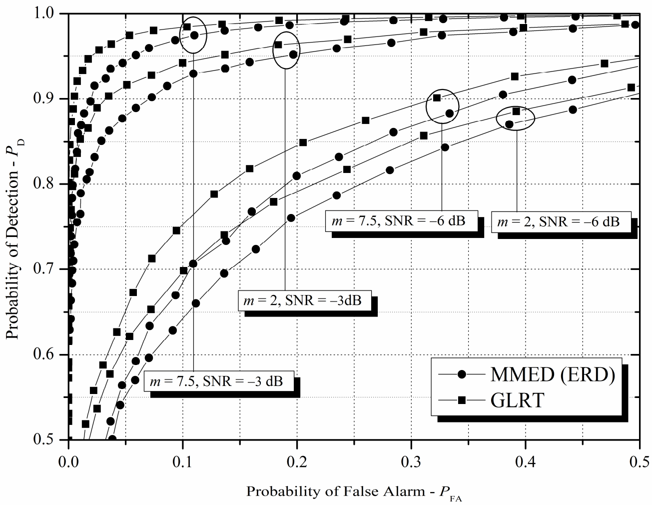

Fig. 2. ROC curves for MMED and GLRT in a Nakagami fading channel under different fading parameter and SNR

In order to show the performance variations as a function of the phase parameter, Figure 3 depicts ROC curves for the MED technique on a Nakagami fading channel, for p = 0, ±0.25 and ±0.5, SNR = –5 dB, n = 50 samples per CR, $\ell = 3$ CRs and m = 1. Due to the lack of space, we present in Figure 3 only results for the MED technique. We attest, however, that very similar behaviors were observed for MMED, GLRT and ED, and that all conclusions drawn from the MED also apply to the other ones. It is interesting to notice that different values of the phase parameter lead to different performances, in spite of having the same Nakagami fading parameter. The effect of changing the phase parameter leads to an imbalance in the quadrature signals. Since that the Nakagami signal is a complex variate, variation in the values of p leads to different performances.

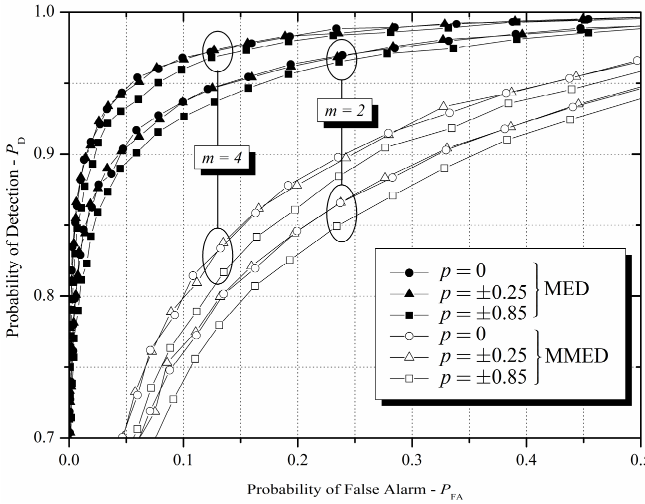

Fig. 3. ROC curves for MED in a Nakagami fading channel with variable phase parameter

The Rayleigh multipath fading is characterized by the absence of line-of-sight or any dominant received signal component between transmitter and receiver, whereas the Rice fading is characterized by the presence of such a received signal component of higher intensity. Then, the effect caused by increasing the Nakagami fading parameter on the improvement of the detection performance is equivalent to the one caused by increasing the Rice parameter. With the objective of illustrating this behavior, Figures 4 and 5 present, respectively, simulation results for known (MED and ED) and unknown (MMED and GLRT) noise variance techniques, using the same settings adopted for constructing Figures 1 and 2, except that at this time a Rice channel is considered, instead of a Nakagami channel. It is evident from these figures the improvement in the detection performance due to an increased Rice parameter, i.e. with an increased strength of a LOS or a dominant received signal component, which decreases fading severity. By setting $K \to 0$ ($-\infty$ dB), the Rice density tends to a Rayleigh density, i.e. the Rice fading turns into a Rayleigh fading. Nevertheless, in [21] it has been pointed out that a Rice parameter around –40 dB suffices to produce a fading that very closely matches a Rayleigh fading. This situation is considered in Figures 4 and 5.

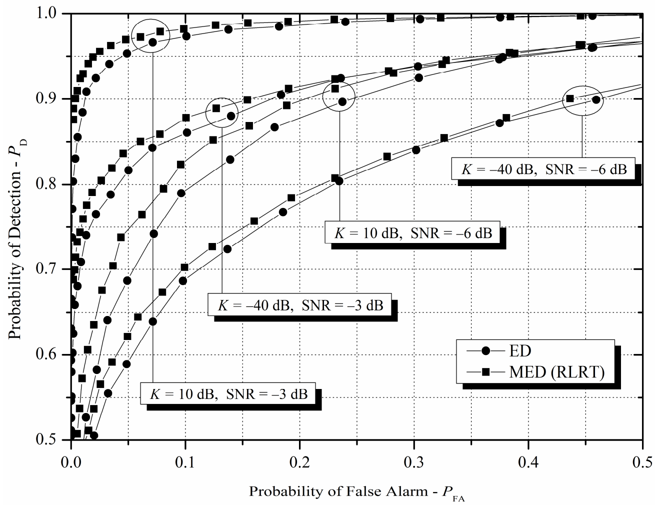

Fig. 4. ROC curves for MED and ED in a Rice fading channel under different Rice factor and SNR

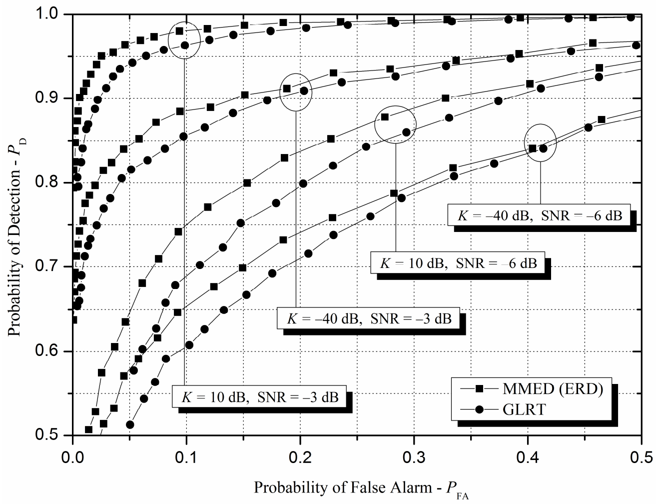

Fig. 5. ROC curves for MMED and GLRT in a Rice fading channel under different Rice factor and SNR

IV. Closing Remarks

This work was devoted to present the results of a performance analysis of the eigenvalue-based generalized likelihood ratio test (GLRT); the maximum-minimum eigenvalue detection (MMED), also known as the eigenvalue ratio detection (ERD); the maximum eigenvalue detection (MED), also known as Roy's largest root test (RLRT); and the energy detection (ED), applied to a centralized data-fusion cooperative spectrum sensing scheme in Nakagami and Rice fading channels. The analysis unveiled significant variations in the sensing performance in terms of variations in Nakagami fading parameter as well as in the phase parameter and Rice parameter. The modeling of the Nakagami channel reflected envelope and phase statistics, which continue to be an interesting debatable question. It was assumed that the channel conditions were modeled with flat fading. For both fading models, it was verified that the MED outperforms all the remaining techniques, followed by ED, GLRT and MMED. The same ranking was also observed in [7]. Since MED and

ED are semi-blind techniques that assume knowledge of noise variance, they achieve better performance than blind ones that do not assume this knowledge, which is the case of GLRT and MMED.

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