Diffusion Constant Modulus Algorithm for Wireless Sensor Networks

Samuel Santos and Aline Neves

***Abstract*— Considering a wireless sensor network in which each node is capable of sensing, computing and communicating with each other, avoiding the need of a fusion center, several distributed algorithms have been proposed in the last years, searching to solve a parameter estimation problem. The large majority of such methods are supervised, needing a training sequence for adaptation. In this paper we propose a blind diffusion algorithm based on the constant modulus algorithm (CMA). We show that the network cooperation enhances the performance of the method when compared to a non-cooperative scheme, and that it exhibits additional robustness by avoiding local minima convergence.**

***Keywords*— Wireless Sensor Network, Constant Modulus Algorithm, Diffusion Algorithms**

I. Introduction

In the last decade, important advances have been made in what concerns signal processing by wireless sensor networks. The possibility of having several sensors working together, each capable of sensing, computing and communicating, enables the improvement of the estimation of unknown system parameters, tracking devices, equalization, among others. The type of network in which nodes cooperate with each other, avoiding the need of a central fusion center, will possibly form the backbone of future generations of data communications, control and sensor networks [1]. Such architecture has shown to be more efficient in terms of energy and communication saving, without mentioning its robustness to nodes failures.

Several methods have been developed for parameter estimation in such context [2]–[5]. The techniques are named *diffusion* or *distributed* algorithms since each node computes its own local estimation, while exchanging information with its neighbors. For parameter estimation, all methods are supervised since a training sequence is necessary for adaptation. However, the availability of such sequence is not always possible, leading to the necessity of developing and using blind techniques.

Interestingly, to the knowledge of the authors, only the work presented in [1] approaches such problem, developing a blind distributed constant modulus based technique for the adaptation of the nodes local estimates. However, the work done in [1] restrains itself to a very particular network: a ring topology, with a single-hop Hamiltonian cycle. In order to generalize such method, so that it can be used and applied to any network topology, in this paper we propose a diffusion constant modulus algorithm (D-CMA). We show, through simulations, that there is a significant performance gain when compared to a non-cooperative scheme. We also briefly analyze the algorithms convergence with respect to the filter initialization, keeping in mind that the constant modulus criterion presents local minima [6]. In addition, the combination matrix, which represents the network topology in the algorithms adaptation, plays an important role in the methods performance, differently from what is observed on supervised methods [2], [3].

This paper is organized as follows. Section II presents the network model used in this work and the existing rules to set the combination matrix. The proposed Diffusion Constant Modulus Algorithm (D-CMA) is explained in section III. Such section also revisits the method proposed in [1]. Section IV discusses the simulation results, and section V concludes this paper.

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II. Network Model and Problem Formulation

In this work, we will consider a network in which nodes are able to exchange information with one another, following the model used in [2], [3]. Figure 1 shows an example of topology. Each sensor measures the outcome of an unknown system. The system is modeled as linear, possibly time-varying filter. The measurement is made in the presence of a phase shift and additive noise that are independent for each sensor [1]:

$$u_k(n) = \sum_{i=0}^{L} e^{j\beta_k} h_i(n) s(n-i) + \nu_k(n) \qquad (1)$$

where $u_k(n)$ is the measurement of the k^{th}-node, β_k is the phase shift for the k^{th}-node, $h_i(n)$ is the filter finite impulse response coefficient, with $i = 0, ..., L$, $s(n)$ is the transmitted signal and $\nu_k(n)$ is the noise for the k^{th}-node. The phase shift β_k is independent for each node k, being modeled as a random variable, uniformly distributed between $[0, 2\pi]$. The noise ν_k is a circular complex white Gaussian noise, with zero mean and variance one.

The objective is to equalize the unknown system, recovering the transmitted sequence $s(n)$. Since we consider a distributed adaptive estimation algorithm, in addition to the measure $u_k(n)$, each node in the network has access to the estimates generated by its neighbors. We will define the neighborhood of a node k as being the set $\mathcal{N}_k$ of all other l-nodes to which k is connected, including itself. Thus, the aggregate estimate ϕ_k is obtained by the fusion of the local estimates of the k^{th}-node neighbors:

$$\boldsymbol{\phi}_k(n) = \sum_{l \in \mathcal{N}_k} c_{k,l} \boldsymbol{\psi}_l(n) \qquad (2)$$

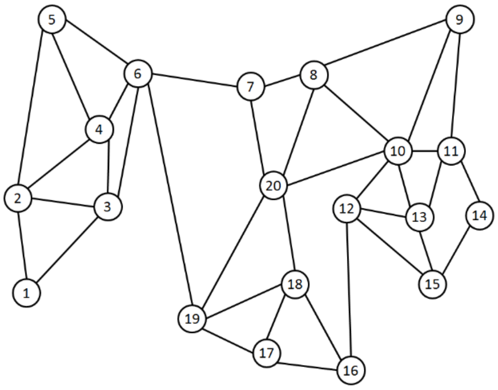

Fig. 1. Network with 20 nodes

where ϕ_k is the aggregate estimate of node k, ψ_l is the local estimate of node l and $c_{k,l}$ are the coefficients that enables the fusion of the local estimates.

The coefficients $c_{k,l}$ are elements of the combination matrix $\mathbf{C}$ [2], [3]. This matrix represents the network topology. As a basic rule, that must be always satisfied, the coefficients related to the neighbors of a node k must always add to one, *i.e.* $\sum_{l\in\mathcal{N}_k} c_{k,l} = 1$ for $k = 1, .., N$, where N is the number of nodes in the network.

Several rules are available to define the matrix $\mathbf{C}$. A few possibilities are briefly described in the sequel:

- *Metropolis rule* (MR) [2], [3], [7]:

$$c_{kl} = \begin{cases} \frac{1}{max\{n_k, n_l\}}, & l \in \mathcal{N}_k \\ 1 - \sum_{l\in\mathcal{N}_k, l\neq k} c_{kl}, & k = l \\ 0, & otherwise \end{cases} \quad (3)$$

where n_k and n_l denote the degree of nodes k and l, *i.e.*, $n_k = |\mathcal{N}_k|$.

- *Laplacian rule* (LR) [2]:

$$C = I_N - \kappa\mathcal{L} \quad (4)$$

where I_N is the identity matrix of order N, $\kappa = 1/n_{max}$ with n_{max} being the largest degree of a node in the network and $\mathcal{L} = \mathcal{D} - \mathcal{A}$ with $\mathcal{D} = diag\{n_1, n_2, ..., n_N\}$ and $\mathcal{A}$ is the $N \times N$ adjacent matrix formed as:

$$[A]_{kl} = \begin{cases} 1, & \text{if } k \text{ and } l \text{ are linked} \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

- *Nearest Neighbor* rule (NN) [2]:

$$c_{kl} = \begin{cases} \frac{1}{|\mathcal{N}_k|}, & l \in \mathcal{N}_k \\ 0, & \text{otherwise} \end{cases} \quad (6)$$

- *Maximum Degree* rule (MD) [8]:

$$c_{kl} = \begin{cases} \frac{1}{N}, & l \in \mathcal{N}_k, \\ 1 - \sum_{l\in\mathcal{N}_k, l\neq k} c_{kl}, & k = l \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

- *Relative Degree* rule (RD) [8]:

$$c_{kl} = \begin{cases} \frac{n_l}{\sum_{m\in\mathcal{N}_k} n_m}, & l \in \mathcal{N}_k, \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

Other recent methods include adaptive combiners searching for the optimization of the network adaptation [4], [5]. Such techniques were proposed in a supervised context. In this work, we will consider a static scenario and thus we will restrain ourselves to the rules presented above, keeping the matrix $\mathbf{C}$ constant during adaptation.

III. Diffusion Constant Modulus Algorithm

To develop a constant modulus algorithm that works in diffusion mode, enabling and exploiting the exchange of information between different nodes, we will follow the basic idea used in [2] where the authors propose a diffusion least-mean squares algorithm, that is, the idea of using not only the local estimate of each node but also an aggregate estimate in which the information of the nodes neighbors are also taken into account (see (2)). It is important however to emphasize that we consider here a completely different scenario, since we want to achieve the blind adaptation of the nodes filters with the objective of equalizing an unknown system, recovering the desired signal $s(n)$.

A. The proposed algorithm

The proposed diffusion constant modulus algorithm (D-CMA) can be stated as follows: each node has an equalizer with output given by:

$$y_k(n) = \mathbf{u}_k(n)\boldsymbol{\phi}_k^H(n) \quad (9)$$

where y_k is the output of the equalizer at the node k, $\mathbf{u}_k(n) = [u_k(n)\ u_k(n-1)\ ...\ u_k(n-M+1)]$ and $\boldsymbol{\phi}_k$ is the vector containing the M coefficients of the aggregate estimate. The superscript H denotes Hermitian transpose. Thus, the equalizer output takes into account not only the local estimate, but also the exchange of information with the nodes neighbors.

The constant modulus criterion searches to minimize the following cost function [9]:

$$J_{CM} = E\big(|y_k(n)|^2 - R_2\big)^2 \quad (10)$$

in which $R_2 = \frac{E|s(n)|^4}{E|s(n)|^2}$, and $E\{.\}$ is the expectation operator.

The gradient-based algorithm for the k^{th}-node can thus be stated as:

$$\boldsymbol{\psi}_k(n+1) = \boldsymbol{\phi}_k(n) - \mu\mathbf{u}_k^H(n)y_k(n)(|y_k(n)|^2 - R_2) \quad (11)$$

where μ is the stepsize.

As we will see in section IV, such an algorithm is more robust, converges faster and attains lower error levels than a non-cooperative scheme.

B. Revisiting the literature

In [1], the authors propose the use of a distributed constant modulus algorithm in a wireless sensor network, in a very particular case: the authors consider a ring network topology with a single-hop Hamiltonian cycle. Therefore, the filter coefficients of the first node are initialized, and the adaptation at each node k only considers the information coming from the precedent node $k-1$.

Comparing to the D-CMA proposed in section III-A, the algorithm proposed by [1] does not have an aggregate estimate, since the topology is fixed, and the output y_k is obtained using the local estimate at node k. Since adaptation is achieved through an incremental procedure, we will name such algorithm as I-CMA. In section IV, we compare the performance of I-CMA and D-CMA, reducing D-CMA to the unique context in which I-CMA may be applied.

IV. Simulation Results

In the following simulations, we considered the transmission of a QPSK (Quadrature Phase Shift Keying) modulated signal, through a channel given by $\boldsymbol{h} = [0.3482 \ -0.8704 \ 0.3482]$. The signal received by the nodes follow (1). We considered $M = 5$ and a center-spike initialization [6]. The performance of the network was measured through the mean residual Intersymbolic Interference (ISI) given by:

$$ISI = \frac{1}{N}\sum_{k=1}^{N} \frac{\sum_i |\alpha_k|^2 - |\alpha_k|^2_{max}}{|\alpha_k|^2_{max}} \qquad (12)$$

where α_k is the combined channel-equalizer (given by the local estimates, $\boldsymbol{\psi_k}$) response for each node k. Figures show the mean of 100 Monte Carlo simulations.

Firstly, we compare the performances of the proposed D-CMA (11) with the I-CMA [1], restraining the system model to a ring network with a single-hop Hamiltonian cycle, since the later only works in such scenario, with $N = 5$. It is interesting to note that the two algorithms work differently: D-CMA takes into account not only the information of the preceding node, as does the I-CMA, but also the information of the current node, due to the definition of the neighborhood $\mathcal{N}_k$. Figure 2 shows the results obtained, showing that, for this specific scenario, the performance of both algorithms is very similar. The stepsizes were $\mu = 0.007$ for D-CMA and $\mu = 0.001$ for I-CMA, and a $SNR = 10$ dB was considered.

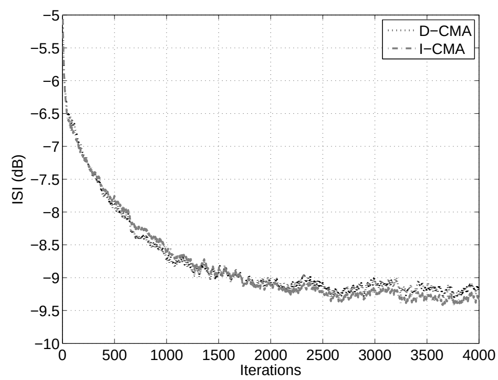

Fig. 2. Comparison between I-CMA and D-CMA on a 5 node ring network

In the sequel, we considered an arbitrary 20 nodes network. Since I-CMA may not be applied to this context, only D-CMA was simulated. Our objective here is to analyze the networks performance when compared to a non-cooperation situation, and also to investigate if the combination matrix $\mathbf{C}$, defined in section II, interferes on the performance of the algorithm. Figure 1 shows the network topology and figure 3 shows the performance of the network for a $SNR = 20$ dB. As expected, the cooperation between nodes results in a faster convergence when compared to a non-cooperative scheme. Stepsizes were adjusted so that the algorithms would converge to the same residual ISI. For D-CMA, $\mu = 0.01$ and $\mathbf{C}$ uses the LR. For CMA with no cooperation, $\mu = 0.03$. It is important to note that here all combination matrix $\mathbf{C}$ led to the same performance.

Figure 4 shows the performance when $SNR = 10$dB. Here we can see how $\mathbf{C}$ interferes on the algorithm. Clearly, LR led to a better performance, followed by MR and RD. Further analysis in this sense are necessary to explain such a difference, keeping in mind that the CMA theoretical analysis is not a simple task since it involves higher-order statistics [6]. Stepsizes used here were $\mu = 0.002$ for CMA with no cooperation, $\mu = 0.003$ for RD, MR, NN and MD, and $\mu = 0.007$ for LR, which showed to be more stable and robust to noise than the others.

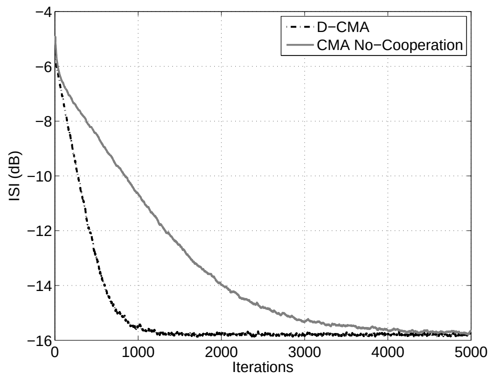

Fig. 3. D-CMA on a 20 nodes network, arbitrary topology, $SNR = 20dB$

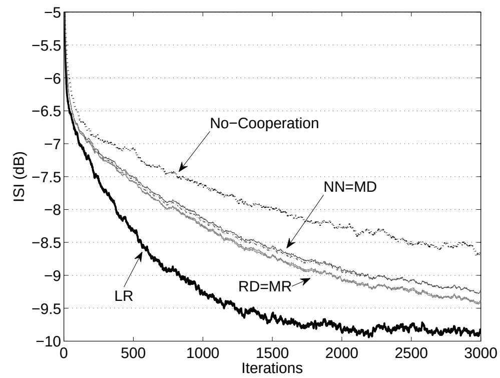

Fig. 4. D-CMA on a 20 nodes network, arbitrary topology, $SNR = 10dB$

Finally, an interesting aspect that arises is with respect to the equalizer initialization. The constant modulus criterion is known for having local solutions that are not capable of reducing ISI, which makes initialization an important step in the process of adaptation [6]. Here, however, we are considering a network. If certain nodes have a bad initialization, will the network be able to overcome such drawback and converge to a good solution? To analyze the convergence of the algorithm,

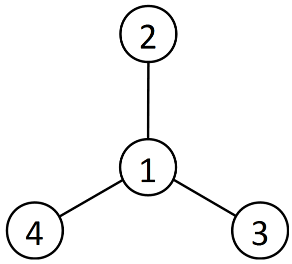

Fig. 5. Network with 4 nodes

Scenario	ψ_{final} MR=LR=MD	ψ_{final} NN	ψ_{final} RD
1 Initialization $\psi_1 = [1\ 0]$ $\psi_{2,3,4} = [0\ 0.8]$	$[0.02\ 0.74]$	$[0.02\ 0.74]$	$[0.02\ 0.74]$
2 Initialization $\psi_{1,3} = [1\ 0]$ $\psi_{2,4} = [0\ 0.8]$	$[0.02\ 0.74]$	$[0.94\ \ -0.35]$	$[0.94\ \ -0.35]$

TABLE I

D-CMA PERFORMANCE FOR DIFFERENT INITIALIZATIONS

we considered a simple two tap channel $h = [1\ 0.4]$, no noise. Using an equalizer with two taps, $\psi = [0.95\ \ -0.32]$ is the CMA global solution and $\psi = [0.04\ 0.75]$ is the local minima, together with their counterparts [10]. We then considered the network shown in figure 5. In the first scenario, we initialized node 1 with $\psi_1 = [1\ \ 0]$, *i.e.*, near the global solution, and all other nodes were initialized with $\psi_k = [0\ 0.8]$, for $k = 2, 3, 4$, *i.e.*, near the local solution. On the second scenario, node 1 and a second node, chosen arbitrarily, were initialized near the best solution while the other two were initialized near the local minima. Stepsize must be small to assure that the algorithm will not escape from local minima. We considered $\mu = 0.01$ for all cases. The results are shown in table 1.

We can see that, having only the central node with a good initialization (first scenario) is not sufficient to achieve the global minima after convergence. On the other hand, if one more node is well initialized (second scenario), a good performance can be achieved, specially using the NN or RD rules. In the given example, second scenario, we chose node 3 to initialize near the global minimum, but the performance will be exactly the same if we choose nodes 2 or 4.

V. CONCLUSION

In this paper we proposed a Diffusion Constant Modulus Algorithm (D-CMA) to be applied in a wireless sensor network of arbitrary topology. We showed that the network performs better with such algorithm than in a non-cooperative scheme. We also showed that, differently from what is observed in similar supervised techniques, D-CMA is sensitive to the rule used to define de algorithms combination matrix, even differing from a local convergence to a global convergence only by the change of such matrix. Further analysis must be undertaken to understand such behavior in more detail.

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