

Semidefinite Relaxation for Large Scale MIMO Detection

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Abstract—The semi-definite relaxation (SDR) is a high performance efficient approach to MIMO detection especially for low modulation orders. We focus on developing a computationally efficient approximation of the maximum likelihood detector (ML) algorithm based on semi-definite programming (SDP) for M -QAM constellations. The detector is based on a convex relaxation of the ML problem. A comparative analysis including the performance-complexity trade-off of the SDR and the lattice reduction (LR) aided linear MIMO detectors considering high number of antennas is carried out aiming to demonstrate the effectiveness of the SDR-based conventional and large scale MIMO detector. SDR-MIMO detector can provide a close, and under high order antennas cases, a better performance than the LR-aided linear MIMO detectors.

Keywords—MIMO detection, lattice reduction, semi-definite relaxation, convex optimization.

I. INTRODUCTION

The multiple-input multiple-output (MIMO) communication systems have arise in many modern communication channels, such as multi-user communication, cooperative networks and multiple antenna channels. It is well known that the use of multiple transmit and receive antennas offers substantial gains to the system in comparison to the traditional single antenna systems. In order to exploit these gains, the system should be able to efficiently detect the transmitted symbols at the receiver side with maximum of energy efficiency and minimal complexity increment.

The optimal detection problem in the sense of minimum joint probability or error for detecting all the symbols simultaneously is solved by the ML detector, which is known as NP-hard [1]. It can be implemented by a brute force-search over all of the possible transmitted vectors set, searching for the one that minimizes the Euclidean distance from the received vector, or using more efficient search algorithms, i.e, the sphere decoder (SD) [1], [2]. However, the expected computational complexity of the ML receiver, even when SD is applied, is unpractical for many channel scenarios and applications. Consequently, there has been much interest in implementing sub-optimal or quasi-optimal MIMO detection algorithms, such as the linear receivers, i.e, the zero-forcing (ZF) and the minimum mean squared error (MMSE) MIMO detectors [1].

One of the most promising quasi-optimal MIMO detection strategies is the semi-definite relaxation (SDR), which provides a better bit error rate (BER) performance than the linear and decision-feedback MIMO receivers [2]–[6] while holds same order of complexity. The SDR attempt to approximate

the solution for the ML problem using a convex program that can be efficiently solved in polynomial time. The usual approach of the SDR problem is first to formulate the ML problem in a higher dimension and then relax the non-convex constraints; such relaxation will result in a semi definite program (SDP), for which there are efficient tools to obtain solutions in polynomial time [7].

SDR was first proposed for signal detection problem on binary/quadratic phase shift keying (BPSK/QPSK) constellations, [3], [5], in which near-ML optimal performances were empirically observed. These results suggest that at high signal-to-noise ratios (SNRs), there is a high probability that SDR will yield the true ML decision. Other result that motivates the use of the SDR shows that many of the other conventional detectors, such as the MMSE, are relaxations of the SDR and are, therefore, inferior in performance ways [6].

The success of SDR in demodulating BPSK signaling motivated its generalization to higher constellations, e.g., the generalization to M -ary quadrature amplitude modulation (M -QAM) signaling was intensively studied in order to conceive high data rate systems. An SDR detector scheme for high-order 16-QAM modulation was proposed in [4], while an approximation of this detector was developed in [8], aiming to achieve a high order 64-QAM constellation signaling. Moreover, in [6], [9] the SDR detection problem is generalized considering 4^q -QAM ($q \geq 1$) modulation orders. In [10] a large scale SDR-based detector is proposed for fast signal detection. The SDR problem is further reduced to the sequential linear programming by adding new form of cutting planes and column generation method. BER performance is compared with linear ZF and MMSE MIMO detectors, as well as the ML optimal detectors for 16×16 and 28×28 antennas. In [11], authors suggest that the conventional SDR detector in a multi-casting problem, where the transmitter is equipped with a massive antenna array, the complexity of solving semi-definite problem (SDP) directly obtained can be prohibitively high. Authors devise the SDP in a dual domain, producing a more computationally efficient solution. Also, they proposed an iterative second-order cone programming solution that is free from employing any randomization step.

This work analyzes the performance-complexity trade-off of the SDR-MIMO detection algorithm, taking as reference both linear sub-optimal and ML optimal solutions; low signaling orders are adopted in the comparison with ML while high order modulation schemes is adopted when comparing with linear ZF or MMSE MIMO detectors.

II. PROBLEM STATEMENT

Considering a standard MIMO channel, the received signal can be described by:

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$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}, \quad (1)$$

where $n_t \times 1$ symbols \mathbf{s} are transmitted simultaneously through a channel which gain is represented by a $n_r \times n_t$ matrix \mathbf{H} and the additive noise $n_r \times 1$ vector samples \mathbf{n} . Each element of the channel matrix \mathbf{H} represents the channel gain in the respective selected path; those gains are assumed known at the receiver side and represented by a Rayleigh distribution. The $n_t \times 1$ vector \mathbf{y} represents the received signal samples in each symbol period, formed by the symbols after passing through the channel. It is also known that the noise vector \mathbf{n} , are samples of additive noise represented as circularly-symmetric Gaussian distribution, $\mathbf{n} \sim \mathcal{CN}\{0, \sigma_n^2 \mathbf{I}\}$, with variance σ_n^2 .

For the subsequent analysis and without loss of generality we assume $n_r = n_t$. The system model is fully defined by complex variables; however, as we are focus on the optimization procedures, for simplicity and computational convenience, the complex variables are split into a double real-value structure. So, rewriting the received MIMO signal in (1) with imaginary and real part separately [1], [12]:

$$\begin{bmatrix} \Re\{\mathbf{y}\} \\ \Im\{\mathbf{y}\} \end{bmatrix} = \begin{bmatrix} \Re\{\mathbf{H}\} & -\Im\{\mathbf{H}\} \\ \Im\{\mathbf{H}\} & \Re\{\mathbf{H}\} \end{bmatrix} \begin{bmatrix} \Re\{\mathbf{s}\} \\ \Im\{\mathbf{s}\} \end{bmatrix} + \begin{bmatrix} \Re\{\mathbf{n}\} \\ \Im\{\mathbf{n}\} \end{bmatrix} \quad (2)$$

We considers a high order M-QAM modulation, where the symbols are denoted by a complex number which real and imaginary part are limited to $\pm(\sqrt{M}-1)$. The structure of the complex set can be represented by $\mathbb{S} = \left\{a + jb \mid a, b \in \left\{-\sqrt{M}-1, -\sqrt{M}+3, \dots, \sqrt{M}-1\right\}\right\}$. For this modulation, the average symbol energy is given by:

$$E_s = \frac{2(M-1)}{3} \quad (3)$$

A. Maximum Likelihood (ML)

The maximum-likelihood (ML) detector performs an exhaustive search over the whole set of possible symbols $s_i \in \mathbb{S}$, in order to decide in favor of the one that minimizes the Euclidean distance between the received signal \mathbf{y} and the reconstructed signal $\mathbf{H}\mathbf{s}$:

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s} \in \mathbb{S}} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2. \quad (4)$$

It is well known that the ML detector provides the lowest BER performance of all MIMO detectors, but the search complexity grows exponentially according to the number of antennas and the number of symbols, leading to a M^{n_T} symbol set combinations.

III. RELAXED ML CRITERION BY SEMIDEFINITE PROGRAMMING

SDR is an efficient approximation tool for non-convex quadratically constrained quadratic programming (QCQP) problems and it has been shown to provide good approximation accuracy in the application of near-ML detection problem with BPSK [3] and QPSK [4], [5] constellations. Like most relaxation methods, SDR consists of three steps: a) relax the feasible set of the original problem in order to ease the solution of the relaxed problem; b) solve the relaxed problem;

c) convert the relaxation solution to an approximate solution of the original problem.

The main idea behind the SDR approach applied to hard decision MIMO detection is to first establish the finite constellation requirement as a low-rank (in this case rank one) constraint on a matrix whose diagonals belong to a finite constellation. After that, those two constrains are relaxed to a positive semi-definite constraint, which makes the resulting problem convex and enables to use semi-definite programming to solve it [13]. More specifically, we can rewrite the ML problem posed in (4) as follows:

$$\begin{aligned} \|\mathbf{y} - \mathbf{H}\mathbf{s}\|^2 &= \mathbf{s}^T \mathbf{H}^T \mathbf{H} \mathbf{s} - 2\mathbf{y}^T \mathbf{H} \mathbf{s} + \|\mathbf{y}\|^2 \\ &= \mathbf{x}^T \mathbf{L} \mathbf{x} + \|\mathbf{y}\|^2, \end{aligned} \quad (5)$$

$$\text{where } \mathbf{L} \triangleq \begin{bmatrix} \mathbf{H}^T \mathbf{H} & -\mathbf{H}^T \mathbf{y} \\ -\mathbf{y}^T \mathbf{H} & 0 \end{bmatrix}$$

Thus the $\hat{\mathbf{s}}$ can equivalently be obtained through

$$\hat{\mathbf{s}} = \underset{\mathbf{s} \in \mathbb{S}}{\operatorname{argmin}} \mathbf{s}^T \mathbf{H}^T \mathbf{H} \mathbf{s} - 2\mathbf{y}^T \mathbf{H} \mathbf{s} \quad (6)$$

since $\|\mathbf{y}\|^2$ does not depend on $\hat{\mathbf{s}}$ [2]. The function of the above problem can equivalently be written as

$$\begin{bmatrix} \mathbf{s}^T & 1 \end{bmatrix} \begin{bmatrix} \mathbf{H}^T \mathbf{H} & -\mathbf{H}^T \mathbf{y} \\ -\mathbf{y}^T \mathbf{H} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{s} \\ 1 \end{bmatrix} \quad (7)$$

and thus by letting $\mathbf{x} = \begin{bmatrix} \hat{\mathbf{s}}^T & 1 \end{bmatrix}^T$ the ML detection problem can be solved examining the equivalent problem in the second line of (5):

$$\begin{aligned} \min_{\mathbf{x} \in \mathbb{R}^{n_t+1}} \quad & \mathbf{x}^T \mathbf{L} \mathbf{x} \\ \text{s.t.} \quad & x_i^2 = 1 \quad i = 1, \dots, 2n_t + 1 \end{aligned} \quad (8)$$

where x_i is the i th component of \mathbf{x} .

Then, SDR utilizes $\mathbf{x}^T \mathbf{L} \mathbf{x} = \operatorname{tr}(\mathbf{x}^T \mathbf{L} \mathbf{x})$ and $\mathbf{X} = \mathbf{x} \mathbf{x}^T$, which lets the MIMO detection problem in (8) for high modulation order be equivalent to

$$\begin{aligned} \min_{\mathbf{X}, \mathbf{x}} \quad & \operatorname{tr}(\mathbf{L} \mathbf{X}) \\ \text{s.t.} \quad & \operatorname{diag}(\mathbf{X}) = \mathbf{e} \\ & \mathbf{X}(2n_t + 1, 2n_t + 1) = 1 \\ & \mathbf{X} \succeq 0; \quad \operatorname{rank}(\mathbf{X}) = 1 \end{aligned} \quad (9)$$

where \mathbf{e} is the vector of all ones.

We should observe that the optimization problem in (9) isn't convex yet* and it is equivalent to (4) in the sense that if the solution of one is known the solution to the second can be easily computed and vice-versa. However, the component that makes (9) hard is more explicit than the constrains in (4). In precisely way, the only difficult constraint in (9) is the rank constraint, $\operatorname{rank}(\mathbf{X}) = 1$, which is non-convex, the objective function and all the other constraints are convex in \mathbf{X} , thus we should drop the rank constraint in order to obtain the relaxed version of the problem (4):

$$\begin{aligned} \min_{\mathbf{X}} \quad & \operatorname{tr}(\mathbf{L} \mathbf{X}) \\ \text{s.t.} \quad & \operatorname{diag}(\mathbf{X}) = \mathbf{e} \\ & \mathbf{X}(2n_t + 1, 2n_t + 1) = 1; \quad \mathbf{X} \succeq 0 \end{aligned} \quad (10)$$

*Because of the rank constraint in \mathbf{X} [2]

where $\mathbf{X} \succeq 0$ means that \mathbf{X} is symmetric and positive semi-definite. The problem (10) is the SDR version for high modulation order of (4) and the difference between them is that the constraints on \mathbf{X} has been replaced by $\mathbf{X} \succeq 0$. The problem in (10) is a semi-definite program and standard methods can be used to solve it in polynomial time [14]. The SDR problems can be handled very conveniently and effectively by readily available (and free) software packages; e.g, by using the convex optimization toolbox CVX [7], we can solve (10) in MATLAB with it's SDP mode.

Moreover, in order to solve a high modulation order problem one constraint must be modified and these modification is denominated as *bound constraint* SDR (BC-SDR). In this work the method for high order modulation problem was based on [8]. The convex optimization problem for high order modulation cases is rewritten on it's relaxed version as:

$$\begin{aligned} \min_{\mathbf{X}} \quad & \text{tr}(\mathbf{L}\mathbf{X}) \\ \text{s.t.} \quad & I_L \mathbf{I} \geq \text{diag}(\mathbf{X}) \geq S_L \mathbf{I} \\ & \mathbf{X}(2n_t + 1, 2n_t + 1) = 1; \quad \mathbf{X} \succeq 0 \end{aligned} \quad (11)$$

where, $I_L = \min \log_2(M)^2$; $S_L = \max \log_2(M)^2$ and \mathbf{I} is the $2n_t + 1$ dimensional identity matrix.

In the backstage, most convex optimization toolboxes handle SDP with an interior point algorithm. Hence, the SDR problem (10) can be solved with a worst case complexity [15]:

$$\mathcal{O} \left(\max \{m, n\}^4 n^{\frac{1}{2}} \log \left(\frac{1}{\epsilon} \right) \right) \quad (12)$$

where, m is the number of constraints, n is the problem size and ϵ is a given solution accuracy. From the point of view of the MIMO equalization problem, the variables m and n are respectively represented by the number of transmit (n_t) and receive (n_r) antennas.

From (12), the SDR complexity scales slowly (logarithmically) with ϵ and most applications do not require a very high solution precision; hence, simply speaking, we can say that the SDR is a computationally efficient approximation approach to QCQP problems, in the sense that its complexity is just polynomial time. So, basically the SDR transforms a NP-hard combinatory problem (4) into a polynomial time solvable problem (10) and (11).

Furthermore, with the relaxation of the rank constraint, a fundamental issue that can be found while using SDR is how to convert a globally optimal solution \mathbf{X}^* of (10) into a feasible solution $\tilde{\mathbf{x}}$ to (4). If \mathbf{X}^* is already rank one, then there is nothing to do, and we can write $\mathbf{X}^* = \mathbf{x}^* \mathbf{x}^{*T}$, and \mathbf{x}^* will be a feasible and optimal solution of (4). On the other hand, if the rank of \mathbf{X}^* is larger than one we must extract from it, in an efficient manner, a vector $\tilde{\mathbf{x}}$ that is feasible for (4) [15].

There are many heuristic ways to extract the rank one solution, however, even though the extracted solution is feasible for (4), it is in general not an optimal solution. Different way to extract the optimal solution from the feasible solution include the rank one approximation and the Gaussian randomization. In this work both rank one approximation and the Gaussian randomization techniques have been deployed.

A. Rank One Approximation

The rank one approximation consists in the most simple technique to extract a solution \mathbf{x}^* to the non-convex problem from the solution of the convex problem formulated, \mathbf{X}^* . With this procedure it is assumed that every solution of \mathbf{X}^* is a rank one solution. Algorithm 1 describes the steps to perform the rank one approximation strategy; in step 3, the operator `slicer(.)` is an approximation to the nearest constellation value.

Algorithm 1 1-Rank Approximation SDR-MIMO Detection

Input: \mathbf{X}^*

Output: \hat{s}_i

- 1: First we should take the eigen-decomposition of \mathbf{X}^*
 $\mathbf{X}^* = \sum_{i=1}^r \lambda_i \mathbf{q}_i \mathbf{q}_i^T$
 - 2: Then we select the higher eigenvalue
 $I = \arg \max_i \lambda_i$
 - 3: Take \mathbf{x}^* as the slicer on the eigenvector constellation associated with the higher eigenvalue.
 $\mathbf{x}^* = \text{slicer}(\mathbf{q}_a)$
 - 4: The estimation of the transmitted symbol in real form is obtained in \mathbf{x}^* , except form the last position of the vector
 $\hat{s}_i = x_i^* \quad i = 1, \dots, 2n_t$
-

B. Gaussian Randomization

The Gaussian randomization procedure is widely deployed; e.g., [15] has demonstrated excellent near-ML results under high number of antennas condition. Alternatively, in this work the Gaussian randomization process based in [4] findings has been used. Algorithm III-B describes such procedure.

Algorithm 2 Gaussian Randomization SDR-MIMO Detection

Input: $\mathbf{X}^*, S_g, \mathbf{L}$,

Output: \hat{s}_i

- 1: Cholesky Factorization at the SDR solution matrix:
 $\mathbf{X}^* = \mathbf{U}^T \mathbf{U}$
 - 2: Let \mathbf{u}_i the i -th column of \mathbf{U}
 - 3: **for** $i = 1$ to S_g **do**
 - 4: Generate a random vector \mathbf{r} with a uniform distributed over a unitary sphere of $(2n_t + 1)$ dimension.
 - 5: Let \mathbf{x}_g be the:
 $\mathbf{x}_g = \text{slicer} \left(\frac{\mathbf{u}_i^T \mathbf{r}}{\|\mathbf{u}_i\|_2} \right), \quad i = 1, 2, \dots, 2n_t + 1$
 - 6: Calculate the the vector \mathbf{k} as:
 $\mathbf{k}_i = \mathbf{x}_g^T \mathbf{L} \mathbf{x}_g, \quad i = 1, 2, \dots, S_g$
 - 7: **end for**
 - 8: $\mathbf{x}_g = \min(\mathbf{k})$
 - 9: $\hat{s}_i = \mathbf{x}_g, \quad i = 1, \dots, 2n_t$
-

IV. NUMERICAL RESULTS

In this section the bit error rate (BER) *versus* E_b/N_0 performance analysis under perfect channel estimation, different number of antennas and modulation order have been considered. The performance and the complexity trade off is an important parameter to be defined; hence, the computation complexity was analyzed for each MIMO detector considered

in this work. Furthermore, we have compared the SDR detector under both estimation approaches with the *lattice reduction zero-forcing* (LR-ZF) strategy. Specifically, on the SDR detection it was utilized the rank one approximation (SDR Rank One) and the Gaussian randomization (SDR Rand) in order to extract the feasible solution \hat{s} from the globally optimum \mathbf{X}^* . Numerical simulations are performed in uncoded spatial multiplexing MIMO systems employing 16-QAM constellations for different antenna configurations, e.g., 8×8 , 16×16 , 64×64 and 128×128 antennas. As demonstrated in the following, the SDR Rand approach overcomes the SDR Rank One approximation for medium/high SNR regions and low size problems. On the other hand, when large MIMO was deployed, an inversion on the BER performance behavior emerges: SDR Rank One MIMO detector overcomes the SDR Rand MIMO detector performance because of its low complexity.

A. BER Performance

Fig. 1 depicts the BER performance for the SDR MIMO detector equipped with both rank approximation and gaussian randomization estimations in comparison with the LR aided linear detectors, namely LR-ZF and LR-MMSE. This procedure was performed in a scenario with 16-QAM constellation, $n_t = n_r = 8$ antennas (Fig. 1.a) and $n_t = n_r = 16$ antennas (Fig. 1.b), under non-line-of-sight (NLOS) Rayleigh propagation channels plus additive white Gaussian noise.

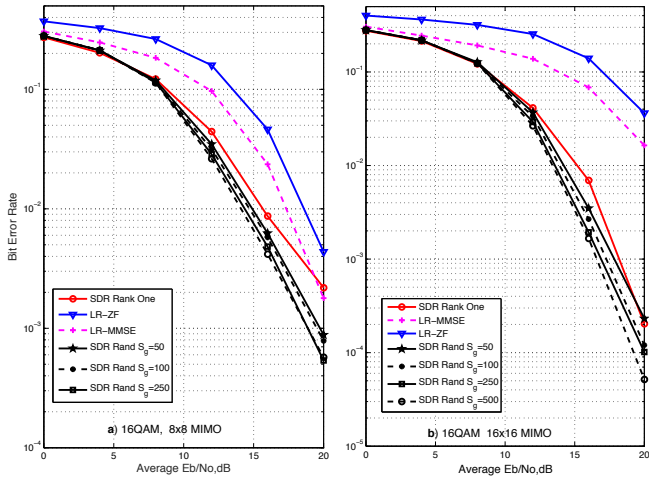


Fig. 1. BER performance for 16-QAM SDR and LR-aided linear MIMO detectors equipped with a) 8×8 antennas and b) 16×16 antennas

Note that both SDR approaches result in better performance under low and high SNR regions; moreover, asymptotically speaking both SDR approximations (Rank One and Rand) tend to get close to each other but at the medium SNR regions the SDR with randomization approach has 4dB gain over the performance of the LR-ZF linear MIMO detector. Fig. 1.b depicts the BER performance for $n_t = n_r = 16$ antennas under the same 16-QAM constellation order and NLOS Rayleigh channel. As the number of antennas grows the LR technique applied to MIMO systems makes them more sensitive to noise, what makes the BER performance be considerable in high SNR regions, where the additive noise is negligible. When the SDR is analyzed a diversity gain was directly observed. Moreover, a considerable performance gain is achieved in high SNR region, something ≈ 7 dB higher.

As a conclusion, the achieved performance of both approximations for the SDR detector in MIMO Rayleigh channels improves progressively with the number of both transmit and receive antennas. Such progressive improvement of SDR Rank One, depicted in Figs. 2.a and 2.b, reflects directly over the complexity the detection strategy. Finally for the Rand approach, as the problem size grows, the number of randomization samples, S_g , must be incremented for better BER performance. Indeed, under lower size problems, the lowest value for S_g on the SDR Rand algorithm have a better BER in comparison to the SDR Rank One approach.

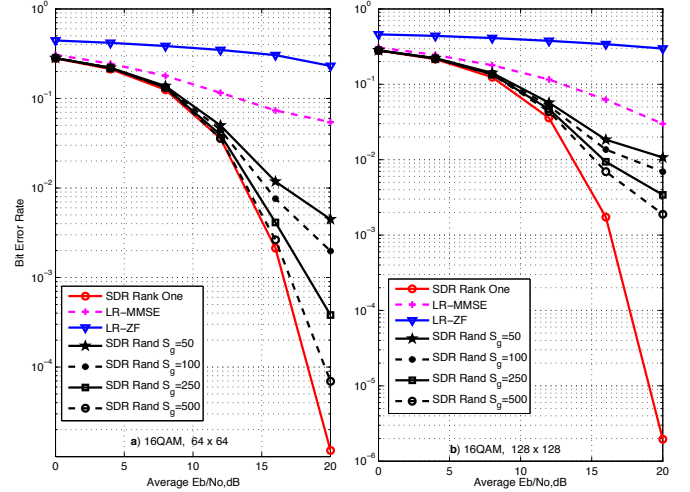


Fig. 2. BER performance for 16-QAM SDR and LR-aided linear MIMO detectors equipped with a) 64×64 antennas and b) 128×128 antennas

B. Complexity

According to [16], the algorithm complexity can be evaluated in terms of the total number of floating-point operations (flops), which one flop is defined as a unitary addition, subtraction, multiplication or division between two floating point numbers. Using this methodology, the complexity of MIMO detectors showed in Table I was determined, where $n_r = n_t$ is the number of receive and transmit antennas, respectively and M is the modulation order in M -QAM constellation.

TABLE I
MIMO DETECTORS COMPLEXITY

Detector	Total Complexity
ML	$M^{n_t}(4n_r n_t + 2n_r)$
LR-ZF	$8n_t^3 + 8n_r n_t^2 + 7n_t^2 + 3n_r n_t - n_r + 5n_t + f_{LLL}(n_t, \rho)$
LR-MMSE	$16n_t^3 + 8n_r n_t^2 + 10n_t^2 + 3n_r n_t - n_r + 4n_t + f_{LLL}(n_t, \rho)$
SDR Rank One	$\frac{16}{3}n_t^3 + 12n_t^2 + \frac{32}{3}n_t + 1$
SDR Rand.	$\frac{16}{3}n_t^3 + 12n_t^2 + \frac{32}{3}n_t + 2 + (8n_t^2 + 26n_t + 10) \cdot S_g$

It was analyzed the complexity for the SDR by evaluating the number of real operations for the rank approximation and the Gaussian randomization, where S_g is the adopted number of generated symbols stored in vector \mathbf{k} , that is used to choose the nearest symbol from the original transmitted one. The computational complexity for the SDR detectors under both estimation techniques are placed near the order of $\mathcal{O}(n_t^3)$, with determines a cubic complexity for the SDR detectors.

It is important to emphasize that the order of constellation doesn't affect the complexity of both SDR algorithms. This

characteristic is achieved by the limitations over the SDR constraints; in the literature it is called *bound constrained* SDR [8]. The specific procedure to determine the SDR detector complexity is detailed in [17] specifying the procedure and the auxiliary packages to perform the analysis.

For the ML approach it is simple verify in Table I that the ML-MIMO detector is highly dependent on the constellation order (problem dimension) what requires a huge number of operations which makes it not feasible even for a low number of antennas. On the other hand, the LR-aided linear MIMO detectors approach the function $f_{LLL}(n_t)$ is an approximation for the flop count on the LLL algorithm presented at the lattice reduction procedure, this function turns out to become more and more complex to solve as the problem when the problem size gets higher which makes the BER for the LR-aided linear equalizer to shown a worst performance in comparison with the SDR approach. Moreover, a surface fitting for the flop count on LLL algorithm was suggested by [12] and described by $f_{LLL}(n_t) = (a + c)n_t^3$, where $a = 5.08 \times 10^{-4}$ and $c = 8.396$. Remembering that this fitting is valid only for $n_r = n_t$ arrays.

The number of complex operations for all those considered MIMO detectors according to the number of antennas and modulation order is depicted in 3D-graphic of Fig 3. The SDR Rand algorithm is highly dependent on S_g which makes the complexity grows as higher as the number of samples. So as the number of antennas grow, the complexity grows proportionally leading to estimation errors. On the other hand, the SDR Rank One approach is suitable for high sized problems leading to the lowest complexity and achieving suitable and the best BER performance for large number of antennas.

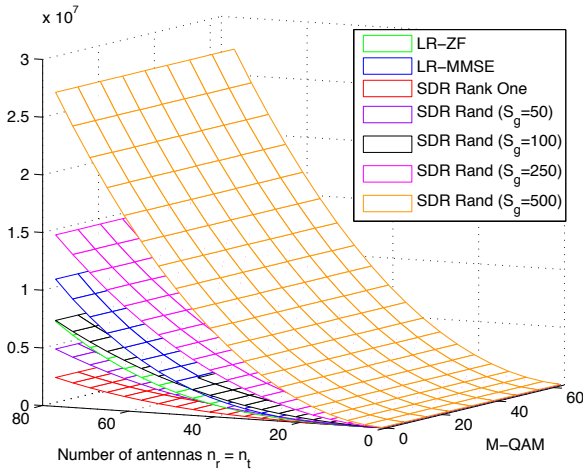


Fig. 3. Complexity of the SDR and LR-aided linear MIMO detectors versus number of antennas and modulation order. For SDR Rand, S_g ranges from 50 to 500.

V. CONCLUSION

Semi-definite relaxation (SDR) technique has been applied to improve the MIMO detection performance in order to achieve near-ML performance on Rayleigh channels. The performance of SDR detectors and their respective computational complexity in term of number of operations under uncorrelated antennas were analyzed. As demonstrated, the SDR-MIMO

detectors outperform the linear techniques, specially when the number of antennas increases. The lattice reduction aided MIMO detectors have an inherent advantage over the most sub-optimal detectors, showing better BER performance over them, the SDR based detector outperform the LR based linear MIMO detectors, specially when the number of antennas increases substantially.

The complexity of the SDR based detectors was reduced by a semi-definite relaxation, which offers similar performance when compared with the conventional LR-aided linear MIMO detectors. As a consequence, the SDR approach presents considerable performance gain with a similar complexity, resulting in a promising solution for high order modulation MIMO systems equipped with a medium-high number of antennas.

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