Damping Factor Analysis on Large Scale MIMO Detector Based on Message Passing

Alex Miyamoto Mussi and Taufik Abrão

***Abstract*— A message passing detector based on belief propagation (BP) algorithm for Markov random field (MRF) graphical model, named MRF-BP, is analysed under large scale MIMO scenarios. The contribution of this work consists of the analysis of message damping method, applied to such MRF-BP detector. A damping factor variation under different number of antennas configuration and SNR regions is considered; BER performance and computational complexity are evaluated over different scenarios. Numerical Results lead to a great performance gain with message damping with no extra computational complexity, although with low number of antennas the damping factor value needs carefully be chosen. Besides, based on the proposed analysis an optimal value for the damping factor is evaluated for different number of antennas scenarios.**

***Keywords*— Low complexity MIMO detector; message passing; damping messages; Markov random fields; graphical models.**

I. INTRODUCTION

Recently, it has been proposed and analyzed communication structures that use tens to hundreds of antennas in transmission and reception of signals, termed *large-scale* MIMO (LS-MIMO) structures, and also called *massive* MIMO or *full dimension* MIMO. Such structures hold the same benefits as conventional MIMO, however, in large-scale. More properly LS-MIMO is defined as a transmission/reception design using typically several tens or even hundreds of antennas in at least one of the communication terminals, usually in the BS [1], [2]. The reduced dimensions of user equipments (UEs) suggests a single antenna arrangement in each UE; on the other hand, a huge amount of the antennas is installed in each BS. In an asymptotic model, an infinite number of antennas at the BS is assumed in a LS-MIMO scheme, resulting in paramount advantages: a) effects of noise background and fast fading channel disappear; b) the transmission rate and the number of UEs become independent of cell size; c) spectral efficiency becomes independent of the system bandwidth; and d) power required for transmission of bit tends to zero [3].

However, these advantages are fully achieved since this unlimited number of antennas at the BS meets a fixed number of UEs. These results become very interesting in scenarios with very erroneous channel estimates due to the high noise power (very low SNR): a sufficient increase in the number of BS antennas in a LS-MIMO system is capable of mitigating harmful effect of error in the channel estimation. On the other hand, in multicellular LS-MIMO systems the use of training pilot sequences for channel estimation purpose imposes a intercellular interference in different cells. The problem is called *pilot contamination*, persisting even in asymptotic BS antennas scenarios [3].

The main advantage achieved with the LS-MIMO scheme refers to high capacity/spectral efficiency [3], [4]; however, with a high number of antennas at BS, the computational complexity of data detection tend to grow proportionally. In this sense, a low computational complexity detector emerges as an essential requirement in LS-MIMO systems, still developing key role to reap the benefits of their high spectral efficiencies. Many low complexity detection procedures for LS-MIMO has been proposed in recent literature, including LS-MIMO detectors based on a) *local neighborhood search*, such as likelihood ascent search (LAS) algorithm [5], and reactive tabu search (RTS) algorithm [6]; b) promising *belief propagation* (BP) based algorithms, such that LS-detectors inspired in graphical models, as *factor graph* (FG) [7] and *Markov random field* (MRF) [8].

BP based detectors have demonstrated a near optimal performance in LS-MIMO scenarios with low computational complexity [9]. Moreover message passing (MP) algorithms based on BP has been reported, in recent literature, as a promising detection procedure in single-carrier spatial modulation LS-MIMO systems [10]. In some situations, BP algorithm may fail to converge, and if it does converge, the estimated marginals may be far from exact [11]. However, there are several methods in the literature to improve the convergence of BP algorithm, including *message damping* method [12], [13] and *double loop* methods [14], [15]. In this work, graphical models and BP procedures are deployed to aid the efficient detection in LS-MIMO systems.

A. Graphical Models

GMs are graphs that indicate inter-dependencies between random variables [16]. Distributions that exhibit some structure can generally be represented naturally and compactly using a graphical model, that is the case of distributions of interest in MIMO systems (e.g., vector of received symbols). The graphical model structure often allows the statistical distribution of interest to be used effectively for inference, i.e., answering certain questions of interest using the distribution [17]. Three basic graphical models widely used to represent statistical distributions include Bayesian belief networks [18], Markov random fields [19], and factor graphs [20].

A MRF is an undirected graph whose vertices are random variables. The variables are such that any variable is independent of all the other variables, given its neighbors. For instance, a V-BLAST MIMO system can be conveniently represented as a MRF with a node for every symbol. Since every transmit antenna is used to transmit a separate symbol, there are N_t nodes in such a graph. Since every transmitted

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symbol interferes with every other transmitted symbol in V-BLAST, the graph is fully connected.

B. Belief Propagation

Belief propagation (BP) is a technique that solves probabilistic inference problems usually implemented in graphical models. BP is a simple, yet highly effective, procedure that has been successfully employed in a variety of applications including computational biology, statistical signal/image processing, data mining, etc. The BP algorithm is now widely recognized an efficient tool that can be used to solve several problems, including communications problems as well [16]. The goal is to detect a hidden input (e.g., transmitted symbol in MIMO systems) from its observed output (e.g., received signal in MIMO systems). The system can be represented as a graphical model and the detection of the system input is equivalent to carry out inference on the corresponding graph. More precisely, BP is a procedure used to compute the marginalization of functions by passing messages on a graphical model [17]. Due to its simplicity and efficiency, BP is the most common strategy adopted to implement message passing principle. Other message passing algorithms can be adressed as *generalized distributive law* (GDL) [21] and sum-product algorithm [20].

Due to the applicability and success of BP in LS-MIMO detection [8], [9], [22], [10], a BP detector based on a MRF graphical model is considered in this work. Furthermore, *message damping* (MD) method is applied and a bit error rate (BER) performance $\times$ damping factor analysis with different antennas configurations in various signal-to-noise ratio (SNR) scenarios is developed in order to evaluate an optimal damping factor value, pointed as the main contribution of this work.

This paper is organized as follow. Section I presents the adopted model for MIMO system, while MRF-based message passing for LS-MIMO detection is discussed in section III. Numerical results are analysed in section IV. Conclusion remarks are provided in section V.

II. System Model

Consider a V-BLAST MIMO communication system with N_t transmit antennas and N_r receive antennas, for simplicity, the channel is assumed to be a frequency-flat fading channel, characterized by the channel matrix $\mathbf{H}$. The elements of $\mathbf{H}$ are all independent complex Gaussian random variables with zero mean and unit variance. Let $\mathbf{x}$ be the $N_t \times 1$ vector corresponding to the BPSK symbols transmitted over the N_t transmit antennas, $x \in \{-1,+1\}^{N_t}$. The additive white Gaussian noise (AWGN) at any receive antenna is assumed to be a complex Gaussian random variable with zero mean and variance σ^2. The matrix model for the system under investigation is

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\eta} \tag{1}$$

where $\mathbf{H}$ represents an $N_r \times N_t$ fading coefficients matrix following a Rayleigh distribution (for amplitudes) representing NLOS (non-line-of-sight) communication and $\boldsymbol{\eta}$ is the noise vector samples with:

$$\mathbb{E}[\eta^2] = N_0 = \sigma^2 = \frac{N_t \varepsilon_{\mathrm{x}}}{\gamma} \tag{2}$$

where ε_{x} denotes the average energy of the transmitted symbols, N_0 is the noise power spectral density which is equal to the variance of $\boldsymbol{\eta}$ entries; and γ is the average SNR per transmit antenna.

III. Message Passing via MRF for LS-MIMO

This section presents a BP based detector that employs message passing on an MRF [9]. Consider the system model in Eq. (1). The maximum *a posteriori* (MAP) detector takes the joint *a posterior* distribution:

$$p(\mathbf{x}|\mathbf{y},\mathbf{H}) \propto p(\mathbf{y}|\mathbf{x},\mathbf{H})p(\mathbf{x}) \tag{3}$$

The MAP estimate of the bit x_i, $i = 1, ..., N_t$ is given by

$$\hat{x_i} = \underset{a\,\in\,\{-1,+1\}}{\arg\max}\; p(x_i = a|\mathbf{y},\mathbf{H}) \tag{4}$$

whose complexity is exponential in N_t [17].

Given $\mathbf{x}$ and $\mathbf{H}$, $\mathbf{y}$ is a complex Gaussian random vector with mean $\mathbf{Hx}$ and covariance $\sigma^2\mathbf{I}_{N_r}$. Thus,

$$p(\mathbf{y}|\mathbf{x},\mathbf{H}) \propto \exp\left(\frac{-||\mathbf{y}-\mathbf{Hx}||^2}{2\sigma^2}\right) \tag{5}$$

Assuming that the symbols in $\mathbf{x}$ are all independent, also necessary for a Markov random fields procedure [17], hence, *a priori* probability for the transmitted symbol is given by

$$p(\mathbf{x}) = \prod_i p(x_i) \tag{6}$$

From Eqs. (3), (5) and (6), the conditional probability function can be written as:

$$\begin{aligned} p(\mathbf{x}|\mathbf{y},\mathbf{H}) &\propto e^{-\frac{1}{2\sigma^2}(\mathbf{y}-\mathbf{Hx})^{\mathrm{H}}(\mathbf{y}-\mathbf{Hx})} \prod_i e^{\ln p(x_i)} \\ &\propto e^{\frac{-1}{2\sigma^2}(\mathbf{x}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\mathbf{Hx} - 2\mathcal{R}\{\mathbf{x}^{\mathrm{H}}\mathbf{H}^{\mathrm{H}}\mathbf{y}\})} \prod_i e^{\ln p(x_i)} \end{aligned} \tag{7}$$

Defining $\mathbf{R} = \left(\frac{1}{\sigma^2}\right)\mathbf{H}^{\mathrm{H}}\mathbf{H}$ and $\mathbf{z} = \left(\frac{1}{\sigma^2}\right)\mathbf{H}^{\mathrm{H}}\mathbf{y}$, Eq. (7) can be rewritten as

$$\begin{aligned} p(\mathbf{x}|\mathbf{y},\mathbf{H}) &\propto e^{-\sum_{i<j}\mathcal{R}\{x_i^* R_{ij} x_j\}} e^{\sum_i \mathcal{R}\{x_i^* z_i\}} \prod_i \exp\left(\ln p(x_i)\right) \\ &= \prod_{i<j} e^{-x_i \mathcal{R}\{R_{ij}\} x_j} \prod_i e^{x_i \mathcal{R}\{z_i\} + \ln p(x_i)} \end{aligned} \tag{8}$$

where z_i and R_{ij} are the elements of $\mathbf{z}$ and $\mathbf{R}$, respectively, and $\mathcal{R}(\cdot)$ denotes the real part of a complex number. Analyzing Eq. (8), it is seen that the MRF of the MIMO system presents pair-wise interactions with the potentials[1] defined by:

$$\psi_{i,j}(x_i, x_j) = \exp\left[-x_i \mathcal{R}\{R_{ij}\} x_j\right] \tag{9}$$

$$\phi_i(x_i) = \exp\left[x_i \mathcal{R}\{z_i\} + \ln p(x_i)\right] \tag{10}$$

The values of ψ and ϕ define, respectively, the edge and self potentials of the MRF graphical model to which message passing algorithm is applied to compute the marginal probabilities of the variables. BP algorithm attempts to estimate the marginal probabilities of all the variables by way of passing messages between the local nodes.

[1]A fully connected subgraph of an MRF is called a clique; the variables in an MRF constrained by a compatibility function is known as a (clique) potential.

A message from node j to node i is denoted by $m_{j,i}(x_i)$, and belief at node i is denoted $b_i(x_i)$, $x_i \in \{\pm 1\}$. The belief $b_i(x_i)$ depends on *how likely* x_i was transmitted. On the other hand, $m_{j,i}(x_i)$ depends on *how likely that node* j *evaluates* x_i *was transmitted*. The message from node i to a neighboring node j is then given by:

$$m_{i,j}(x_j) = \sum_{x_i} \phi_i(x_i)\psi_{i,j}(x_i, x_j) \prod_{k\in\mathcal{N}(i)\backslash j} m_{k,i}(x_i) \quad (11)$$

where $\mathcal{N}(i)$ denotes the set of all nodes neighboring the node i and $\mathcal{N}(i)\backslash j$ denotes the same neighborhood, except the node j. Eq. (11) actually constitutes an iteration, as the message is defined in terms of the messages from other nodes. So, BP essentially involves computing the outgoing messages from a node to each of its neighbors using the local joint compatibility function and the incoming messages and transmitting them [17]. The algorithm terminates after a fixed number of iterations.

A. Message Damping

The MD method can be used to improve the convergence of BP algorithm. The messages to be passed are computed as a weighted average of the message in the previous iteration and the message in the current iteration [12], [13]. Thus, the damped message to be passed from node i to node j in iteration t, denoted by $m_{i,j}^{(t)}(x_j)$, is computed as a convex combination of the previous message and the current message as:

$$m_{i,j}^{(t)}(x_j) = \alpha m_{i,j}^{(t-1)}(x_j) + (1-\alpha)\tilde{m}_{i,j}^{(t)}(x_i) \quad (12)$$

where $\tilde{m}_{i,j}^{(t)}(x_j)$ and $m_{i,j}^{(t-1)}(x_j)$ denotes, respectively, the current message in iteration t and the previous message in iteration $t-1$, and $\alpha \in (0, 1]$ is referred as the damping factor (DF). This simple damping of messages has been shown to be very effective in improving BP convergence and performance [12]. As shown in numerical results section, considering the LS-MIMO detection context, message damping method can improve performance significantly, without increasing the computational complexity. Furthermore, the analysis of damping factor in different LS-MIMO configurations is the main contribution of this work.

Damping of messages can be carried out in each iteration. The final belief about the variable x_i is computed as:

$$b_i(x_i) \propto \phi_i(x_i) \prod_{j\in\mathcal{N}(i)} m_{j,i}(x_j) \quad (13)$$

In the case of a coded system, the soft output of the algorithm $b_i(x_i)$ can be directly fed to the decoder. A pseudocode for the MRF BP described above is listed in Algorithm 1.

IV. Simulation Results

In this section the uncoded bit error rate (BER) performance related to the MRF-BP algorithm for LS-MIMO detection is evaluated through Monte Carlo simulations. The simulations are performed for a V-BLAST MIMO configuration and assuming that a perfect channel state information (CSI) is available at the receiver side. For comparison purpose, the BER performance of a single-input single-output (SISO) transmission scheme operating in flat fading, as well as in purely AWGN channels were included in several graphs. Table I summarizes the main system and channel parameter values deployed in this section. Also, the number of transmit antennas per mobile user in all scenarios is fixed in $N_{tu} = 1$ antenna

Algorithm 1 MRF BP for LS-MIMO detection

Initialization
$m_{i,j}^{(0)}(x_j) = 0.5$, $p(x_i = \pm 1) = 0.5$, $\forall i, j = 1, \ldots, N_t$
$\tilde{m}_{i,j}^{(0)}(x_j) = 0.5$, $\forall i, j = 1, \ldots, N_t$
$\mathbf{R} = \left(\frac{1}{\sigma^2}\right)\mathbf{H}^{\mathrm{H}}\mathbf{H}$; $\mathbf{z} = \left(\frac{1}{\sigma^2}\right)\mathbf{H}^{\mathrm{H}}\mathbf{y}$
for $i = 1$ **to** N_t **do**
  $\phi_i(x_i) = \exp\left(x_i \mathcal{R}\{z_i\} + \ln(p(x_i))\right)$
end for
for $i = 1$ **to** N_t **do**
  for $j = 1$ **to** N_t, $j \neq i$ **do**
    $\psi_{i,j}(x_i, x_j) = \exp\left(-x_i \mathcal{R}\{R_{i,j}\} x_j\right)$
  end for
end for
Iterative update of messages
for $t = 1$ **to** $\mathcal{I}$ **do**
  Message calculation
  for $i = 1$ **to** N_t **do**
    for $j = 1$ **to** N_t, $j \neq i$ **do**
      $\tilde{m}_{i,j}^{(t)}(x_j) \propto \sum_{x_i} \phi_i(x_i)\psi_{i,j}(x_i, x_j) \prod_{k\in\mathcal{N}(i)\backslash j} m_{k,i}^{(t-1)}(x_i)$
      Damping messages
      $m_{i,j}^{'(t)}(x_j) = \alpha m_{i,j}^{(t-1)}(x_j) + (1-\alpha)\tilde{m}_{i,j}^{(t)}(x_j)$
      Messages normalization
      $m_{i,j}^{(t)}(x_j) = \frac{m_{i,j}^{'(t)}(x_j)}{\sum_{x_j} m_{i,j}^{'(t)}(x_j)}$
    end for
  end for
end for
Belief calculation
for $i = 1$ **to** N_t **do**
  $b_i(x_i) \propto \phi_i(x_i) \prod_{j\in\mathcal{N}(i)} m_{j,i}^{(\mathcal{I})}(x_i)$
end for
Detection of data bits
$\hat{x_i} = \arg\max_{x_i\in\{\pm 1\}} b_i(x_i)$, $\forall i, j = 1, \ldots, N_t$
Terminate

In Fig. 1 the BER performance of the implemented MRF-BP algorithm is showed for various $N_t = N_r = U$ antennas configurations. Moreover, a number of iterations $\mathcal{I} = 4$ in BP algorithm were adopted. From Fig. 1.a), one can conclude that under few BP iterations, the LS-MIMO MRF-BP performance tends to the SISO AWGN system performance when the number of antennas increasing, achieving such performance in medium SNR when $N_t = 500$ antennas. Indeed, with 500×500 antennas configuration, the BER performance and diversity gain achieved by MRF-BP detector is very close to that of SISO AWGN bound leading to a conclusion that, in such large antennas configuration, the performance of MRF-BP detector reaches the asymptotic LS condition and an increasing in $N_t = N_r$ would not result in a relevant performance gain. From these results, it can be concluded that the MRF-BP detector demonstrates promising BER performance in large-scale MIMO systems. Fig. 1.b) shows the MRF-BP detector with optimal DF value, evaluated from numerical results presented in Figs. 3 and 4. The performance and diversity gain with damping messages is noticeable in all antenna configuration.

Fig. 2 evidences the influence of the number of iterations ($\mathcal{I}$) of BP algorithm on the BER performance at $\text{SNR} = 10\text{dB}$ without damping messages, i.e., adopting $\alpha = 0$. One can notice the occurrence of a significant performance gain in the first four iterations of Algorithm 1. Besides, with the increasing number of antennas, the performance of MRF-BP detector improves accordingly. For all antennas configuration depicted in this graph, the performance gain occurs, mainly, in the first four iterations, after that, there is no relevant performance gain, i.e., no relevant information is carried out in neighborhood messages.

TABLE I

LS-MIMO SYSTEM AND CHANNEL PARAMETERS

Parameter	Value
Link direction	Uplink (UL)
# Rx antennas	$N_r \in [20; 50; 100; 500]$
# Mobile users	$U = N_r$
# Tx antennas (per user)	$N_{tu} = 1$
# Tx antennas (total)	$N_t = U.N_{tu}$
Channel type	flat Rayleigh
Channel availability	Perfectly known at receiver
Modulation order	BPSK
# iterations MRF-BP	$\mathcal{I} \in \{4, 5\}$

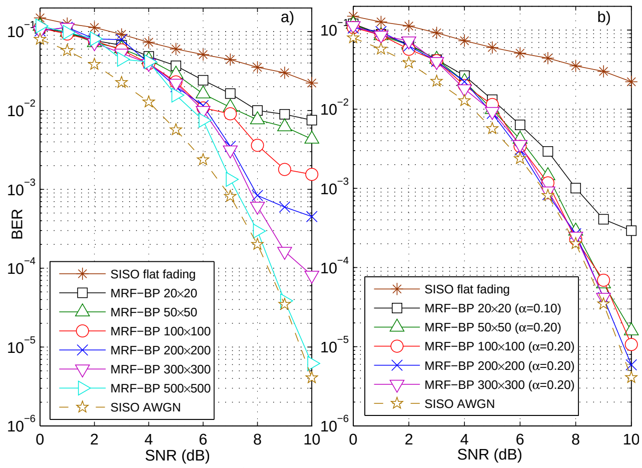

Fig. 1. BER performance of the implemented MRF-BP detector with various $N_t = N_r$ antennas and 4 BP iterations; a) no dampling messages and b) with optimal DF value.

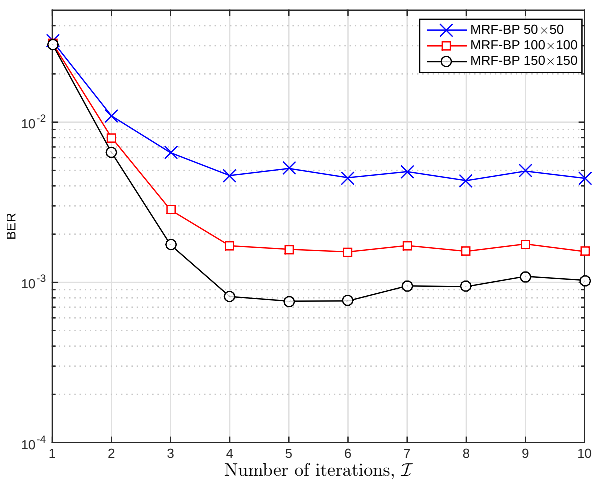

Fig. 2. BP iterations effect on the BER performance; $\text{SNR} = 10\text{dB}$.

Figs. 3 shows the message damping impact on BER performance considering medium-high SNR regions. A MRF-BP detector with message damping variation in the range $0 \leq \alpha \leq 1$ is considered for *a)* 20×20 and *b)* 50×50 antennas. For both antennas configuration, one can notice that, the performance gain with message damping increases in higher SNR regions; in SNR equal to 6dB the performance gain with damping ($\alpha \neq 0$) is less than one decade; on the other hand, in the scenario with SNR= 12dB the performance gain with message damping is approximately 2 and 3 decades for $N_t = 20$ and 50 antennas, respectively. Specific conclusions regarding the scenario *a)* $N_t = 20$ antennas is that in higher SNR region, the best damping factor, i.e., associated with the lowest BER, has a lower value and tends be more responsive to α variations. Looking at the implicit curve of SNR= 6dB, the performance variation from $0.25 \leq \alpha \leq 0.45$ is negligible; in the case of 14dB, the BER performance degradation from $\alpha = 0.05$ to 0.20 is more than one decade. Furthermore, the performance becomes worse regarding no damping messages case ($\alpha = 0$) from $\alpha \geq 0.35$ in SNR= 14dB; the same situation occurs from $\alpha \geq 0.80$ when SNR= 6dB. Therefore, the DF value needs to be accurately chosen, specially in medium/high SNR regions.

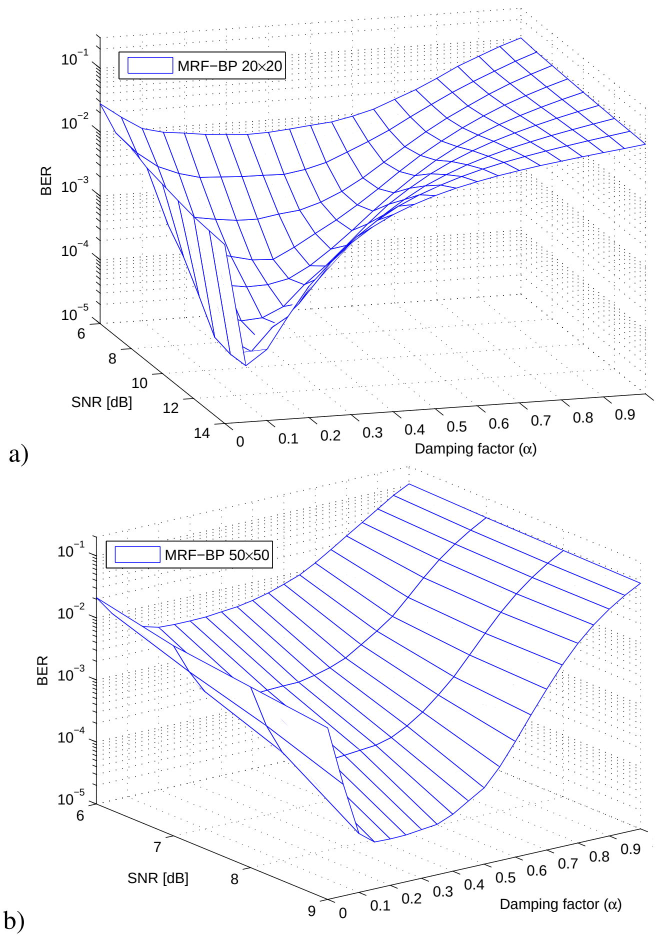

Fig. 3. MRF-BP LS-MIMO performance, $\mathcal{I} = 5$, as a function of different SNR scenarios and damping factor α: **a)** 20×20; **b)** 50×50 antennas

From Fig. 4, it is distinguishable that, when N_t of LS-MIMO system increases, e.g., from 20×20 to 100×100 antennas, the BER performance behaviour with damping factor demonstrates constant (flat condition) at greater intervals of α.

The same way that, with 50×50 antennas the performance is flatter than 20×20 case (Fig. 3.a and 3.b). One can conclude that, the difficult task to accurately choose the DF value in small number of LS-MIMO antennas scenario, i.e., 20×20, is relaxed with increasing number of antennas. Thus, the application of message damping, specially in high SNR region, is more suitable in LS-MIMO systems due to the flat BER performance response to DF variation.

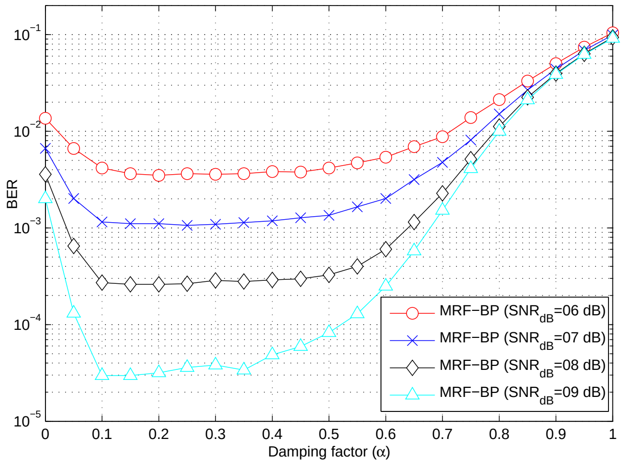

Fig. 4. Variation of damping factor on the BER performance of a MRF-BP 100×100 considering different SNR scenarios; $\mathcal{I} = 5$.

A. Computational Complexity of MRF-BP LS-MIMO detector

The computational complexity is described in terms of floating-point operations, which one floating-point operation denotes the computational complexity of the real mathematical operations: addition, subtraction, multiplication or division. Table II describes the per-symbol computational complexity involved in each step of MRF-BP algorithm. The overall per-symbol complexity of the Algorithm 1 is about $\mathcal{O}(N_t^2)$. It is important to note that *message damping does not increase the computational complexity* of the MRF-BP algorithm.

TABLE II
COMPUTATIONAL COMPLEXITY OF MRF-BP ALGORITHM.

Procedure	Algoritm 1	Per-symbol Complexity
$\mathbf{R}$	line 4	$N_r - 1$
$\mathbf{z}$	line 4	$N_r - 1$
ϕ, eq. (9)	line 6	4
ψ, eq. (10)	line 10	$12N_t$
Messages update	lines 15 to 25	$(4N_t^2 + 4N_t - 8) \cdot \mathcal{I}$
Message damping	line 20	$\frac{4}{N_t}$ *(negligible)*

V. CONCLUSIONS

A detector for LS-MIMO systems based on message passing MRF graphical model and BP algorithm was analyzed. More specifically, message damping method impact on the BER performance was evaluated and has demonstrated a promising method for message passing detectors, specially in LS-MIMO scenarios. Numerical results for the MRF-BP LS-MIMO detector has demonstrated promising performance $\times$ complexity tradeoff, since damping messages procedure just increases marginally the overall computational complexity while providing a significant performance gain. Besides, under large scale antenna scenarios, the DF value choose is facilitated due to the flat BER performance response to α variation.

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