Sensing-Throughput Tradeoff in Cognitive Radio Network

Aislan Gabriel Hernandes and Taufik Abrão

Abstract— **This paper deals with the relationship between the sensing time and throughput in a cognitive radio network (CRN) using energy detector (ED). The sensing time is a period in the medium access control (MAC) protocol that the secondary user (SU) spent to sensing the spectrum. This parameter is critical to determine the performance of the SU and the interference to primary user (PU). In cognitive radio (CR), increasing the sensing time is equivalent to increase the SU performance; accordingly, the throughput of the SU decrease, reducing the SU quality of service (QoS). Such configuration set a tradeoff between sensing time and throughput. In this contribution, the sensing-throughput optimization (STO) problem is formulated to deal with such tradeoff, where the throughput of the SU is maximized. The STO resulting in a convex nonlinear optimization problem (NLP) that can be solved using efficient solvers. Numerical analysis examines the performance of the ED when the throughput is maximized, as well when distinct parameters values are assumed.**

Keywords— **Cognitive Radio Network, Secondary User, Primary User, Spectrum Sensing, Energy Detection, Throughput Maximization.**

I. Introduction

Due to the growth of the wireless communication services, the available spectrum has become scarce. Measurements carried out by Federal Communications Commission (FCC) have demonstrated that the most part of the allocated spectrum is not utilized [1]. The period of time of the spectrum occupancy varies from milliseconds to hours. This motivates the use of the cognitive radio (CR) [2], [3] which is able to increase the spectrum efficiency (SE) considerably. In a wireless regional area networks (WRANs), the main objective is to maximize the spectrum utilization of the TV channels. The CR is the main technology in the WRAN 802.22, where each medium access control (MAC) frame consists of one sensing slot and one data transmission slot. The sensing duration strongly impacts the network throughput. If sensing duration increases the throughput decreases. However, a longer sensing time improves the detection performance, i.e, the SU become more aware about the received signal while more protection is given to the PU.

Recently, more and more importance has been given to the sensing time *vs.* throughput tradeoff in the context of CRN [4], [5]. The scheme to deal with this challenging issue consists in formulate and efficiently solve the associated optimization problem that maximize the throughput subject to different constraints, such as probability of detection, probability of false alarm, maximum frame time, optimum threshold, and so forth. For instance, in [6], [7], the sensing-throughput optimization problem in ED spectrum sensing was formulated as convex nonlinear optimization problems. Moreover, in [8], [9] the non-cooperative double threshold spectrum sensing was analysed in the sensing-throughput tradeoff perspective. In [10], the multichannel cooperative sensing optimization problem was formulated as a nonconvex mixed-integer problem that is solved dividing the original problem into convex mixed-integer subproblems.

In [11], a convex nonlinear optimization problem is formulated to deal with the STO tradeoff in a singleband cognitive radio network. In this sense, this contribution consists in a numerical analysis extension of the work in [11]. The STO problem was solved expeditiously using the solver available into the *Matlab Optimization Toolbox*. Simulation results demonstrate the quality of solution and the impact in the ED performance and some interesting results are discussed when the parameters of simulation are different to the initial problem considerations, revealing the ED performance dependence with such parameters.

The rest of the paper is organized as follow. The CR system model is summarized in section II. The formulation of the sensing time *vs.* throughput optimization problem is developed in section III. Numerical results supporting our finding are discussed in section IV. Finally concluding remarks are offered in section V.

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II. System Model

The transmitted PU signal samples is represented by $s(i)$ while a circular symmetric complex Gaussian (CSCG) noise samples are represented by $n(i)$. The received signal at the SU is written by

$$y(i) = s(i) + n(i), \qquad i = 1, 2, ..., \tau f_s. \tag{1}$$

where τf_s is the total number of samples, τ is the sensing time and f_s is the sampling frequency.

A binary decision hypothesis is taken if the channel is idle or busy, respectively, as

$$\begin{cases} \mathcal{H}^0 : y(i) = n(i), & i = 1, 2, ..., \tau f_s. \\ \mathcal{H}^1 : y(i) = s(i) + n(i), & i = 1, 2, ..., \tau f_s. \end{cases} \tag{2}$$

where $\mathcal{H}^0$ and $\mathcal{H}^1$ are the hypothesis of the absence and presence of the primary user.

The basic MAC frame time structure considered herein is depicted in the Fig. 1. The first portion of the frame time is used to sensing the spectrum and the second portion is related with the transmission time, that impacts in the throughput. Considered that the total frame time is fixed, then the sensing time and throughput are conflicting parameters.

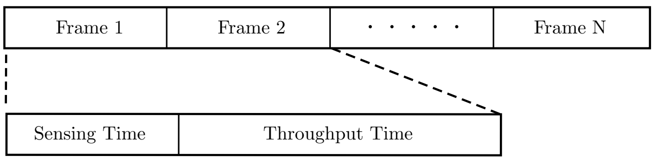

Fig. 1. Sensing-throughput frame time structure.

A. Energy Detector

The ED is the more simple form to spectrum sensing in CRN. It simply estimates the energy content in a determined spectrum bandwidth. The associated statistical test is formulated as

$$T(y) = \frac{1}{\tau f_s} \sum_{i=1}^{\tau f_s} |y(i)|^2. \tag{3}$$

Such statistical test is compared with a threshold level

$$T(y) \underset{\mathcal{H}^0}{\overset{\mathcal{H}^1}{\gtrless}} \lambda, \tag{4}$$

if the statistical test is smaller than threshold level λ, the SU chooses as a idle channel, otherwise the channel is busy and the SU will not transmit.

There are four scenarios that must be considered in the ED performance analysis:

1) If the channel is idle and the SU estimates that the channel is idle, then the SU will transmit and the throughput is maximum. A *correct detection* occurs;
2) If the channel is idle and the SU estimates that the channel is busy, then the SU will not transmit and a *false alarm* occurs;
3) If the channel is busy and the SU estimates that the channel is idle, then the SU will transmit and a *miss detection* occurs;
4) Finally, if the channel is busy and the SU estimates that the channel is busy, then the SU will not transmit and the PU is protected. A *correct detection* occurs.

In this work, the interest is concentrated on the first and third scenarios, *correct detection* and *miss detection* respectively.

III. Sensing-Time *vs.* Throughput Problem Formulation

The probability of false alarm $P_f(\cdot)$ and probability of detection $P_d(\cdot)$ associated to the ED can be formulated using the *central limit theorem* (CLT) approach, as function of sensing time parameter τ

$$P_f(\tau) = Q\left(\sqrt{2\text{SNR}_p + 1}Q^{-1}(\overline{P_d}) + \sqrt{\tau f_s}\text{SNR}_p\right), \tag{5}$$

$$P_d(\tau) = Q\left(\frac{1}{\sqrt{2\text{SNR}_p + 1}}Q^{-1}(\overline{P_f}) - \sqrt{\tau f_s}\text{SNR}_p\right), \tag{6}$$

where $\overline{P_d}$ and $\overline{P_f}$ are the probability of detection target and false alarm target, respectively, and the integral the of Gaussian probability density function is defined as

$$Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(\frac{-z^2}{2}\right) dz. \tag{7}$$

The value of the threshold λ can be related with the probability of detection $P_d(\tau)$ as [11]

$$P_d(\tau) = Q\left((\lambda - \text{SNR}_p - 1)\sqrt{\frac{\tau f_s}{2\text{SNR}_p + 1}}\right). \tag{8}$$

When different values of probability of channel occupancy occurs, then eq. (8) can be extended to:

$$P_d(\tau) = Q\left((\lambda - \beta - 1)\sqrt{\frac{\tau f_s}{2\beta + 1}}\right), \tag{9}$$

where $\beta = \text{P}_\text{r}(\mathcal{H}^1)\text{SNR}_p$. The value of SNR_p is weighted to $\text{P}_\text{r}(\mathcal{H}^1)$, which is the probability of the channel be busy.

The signal-to-noise ratio (SNR) of the primary user signal received in the primary user is given by $\text{SNR}_p = \frac{P_p}{N_0}$ and SNR of the secondary user signal is given by $\text{SNR}_s = \frac{P_s}{N_0}$, where P_p and P_s are the transmission power of the PU and the SU respectively, while the same level of the noise power spectral density N_0 is assumed for both PU and SU user types.

As a consequence, the throughput of the SU in the absence and in the presence of the PU are given respectively by

$$C_0 = \log_2(1 + \text{SNR}_s), \tag{10}$$

$$C_1 = \log_2\left(1 + \frac{\text{SNR}_s}{1 + \text{SNR}_p}\right), \tag{11}$$

where C_0 is the throughput of the SU when it operates in the absence of the PU and C_1 is the throughput of the SU when it operates in the presence of the PU. Obviously, the value of C_0 is always larger than the value of the C_1, i.e the throughput when the channel is busy suffers interference from the PU signal. Therefore, the first and third scenarios lead to the sensing-throughput relations [11]

$$B_0(\tau) = \frac{T - \tau}{T} C_0, \tag{12}$$

$$B_1(\tau) = \frac{T - \tau}{T} C_1. \tag{13}$$

In the first case, the PU is not present then SU not generate false alarm. For the second case PU signal is active. Hence, $B_0(\tau)$ and $B_1(\tau)$ represent the SU throughput dependent on the sensing-time duration ($\tau < T$) when PU is absent and present, respectively.

The probabilities for occurrence of the first and third scenarios are given by [11]

$$\text{P}_\text{r}(\text{correct detection}) = [1 - P_f(\tau)] \cdot \text{P}_\text{r}(\mathcal{H}^0), \tag{14}$$

$$\text{P}_\text{r}(\text{miss detection}) = [1 - P_d(\tau)] \cdot \text{P}_\text{r}(\mathcal{H}^1), \tag{15}$$

where $\text{P}_\text{r}(\mathcal{H}^0)$ and $\text{P}_\text{r}(\mathcal{H}^1)$ is the probability of the channel is idle and busy (related to the first and third scenarios), respectively. The probability $(1 - P_d(\tau))$ is called miss detection probability.

So, the throughput $R_0(\tau)$ and $R_1(\tau)$ for the first and third scenarios are respectively

$$R_0(\tau) = \frac{T - \tau}{T} C_0 \cdot [1 - P_f(\tau)] \cdot \text{P}_\text{r}(\mathcal{H}^0), \tag{16}$$

$$R_1(\tau) = \frac{T - \tau}{T} C_1 \cdot [1 - P_d(\tau)] \cdot \text{P}_\text{r}(\mathcal{H}^1). \tag{17}$$

Finally, the total throughput in the SU network is given by

$$R(\tau) = R_0(\tau) + R_1(\tau). \tag{18}$$

For the case of the ED spectrum sensing, the throughput is given by eq. (19) [11], at the top of next page.

To simplify, we consider that the probability of the channel is occupied is low, i.e $\text{Pr}(\mathcal{H}^1) \leq 0.2$ and the second term of the throughput function in (19) becomes insignificant and can be simplified as

$$\widehat{R}(\tau) = B_0(\tau)\left(1 - Q\left(\sqrt{2\text{SNR}_p + 1}Q^{-1}(\overline{P_d}) + \right.\right.$$
$$\left.\left. + \sqrt{\tau f_s}\text{SNR}_p\right)\right) \text{P}_\text{r}(\mathcal{H}^0), \tag{20}$$

Finally, the simplified sensing-throughput optimization (STO) problem can be expressed as

$$\begin{array}{lll} \underset{\tau}{\texttt{max.}} & \widehat{R}(\tau) & \\ \texttt{s.t.} & \texttt{(C.1.)} & 0 \leq \tau \leq T \\ & \texttt{(C.2.)} & P_d(\tau) \geq \overline{P_d} \end{array} \tag{21}$$

where $\overline{P_d} = 0.9$ is the probability of detection target according to the IEEE 802.22 WRAN. The convexity of the optimization problem (21) is demonstrated in the Appendix.

The optimization problem above can be interpreted as a sensing-throughput tradeoff whose objective is to identify the optimal sensing duration τ for each frame time in the MAC layer, such that the achievable throughput of the SU is guaranteed, while ensure the PU protection, that is related with the value of the P_d.

IV. Numerical Results

Table I depicts the main parameter values deployed in this section. The values of the throughput using such parameters are $C_0 = 6.6582$ and $C_1 = 6.6137$.

TABLE I

Reference values used for simulations.

Parameter	Value
$\text{Pr}(\mathcal{H}^0)$	[0.8, 0.5, 0.2]
$\text{Pr}(\mathcal{H}^1)$	[0.2, 0.5, 0.8]
SNR_s	20[dB]
SNR_p	-15[dB]
T	100[ms]
$\overline{P_d}$	0.9
f_s	6[MHz]
PU signal	QPSK

Using the simple but effective tool *fmincon* of *MATLAB Optimization Toolbox*, the STO problem was solved easily and the solver returns the optimal sensing time value equal to $\tau^* = 2.6$ [ms] for the three scenarios, i.e., for low, medium as well as high channel occupancy; the estimated optimal throughput $\widehat{R}^*$, original optimal throughput and the difference are given in Table II.

Since the number of samples N_s is related to the sensing time and the sample frequency and considering that the optimum sensing time for the three scenarios results same, then

$$N_s^* = \tau^* f_s = 15600 \quad \text{[samples]}. \tag{22}$$

$$R(\tau) = B_0(\tau)\left(1 - Q\left(\sqrt{2\mathrm{SNR}_p + 1}Q^{-1}(\overline{P_d}) + \sqrt{\tau f_s}\mathrm{SNR}_p\right)\right)\mathrm{P_r}(\mathcal{H}^0) + \\ + B_1(\tau)\left(1 - Q\left(\frac{1}{\sqrt{2\mathrm{SNR}_p + 1}}Q^{-1}(\overline{P_f}) - \sqrt{\tau f_s}\mathrm{SNR}_p\right)\right)\mathrm{P_r}(\mathcal{H}^1). \tag{19}$$

TABLE II

SIMPLIFIED, ORIGINAL AND DIFFERENCE OF THROUGHPUT.

$\Pr(\mathcal{H}^1)$	0.2	0.5	0.8
$\widehat{R}^*$ $\left[\frac{\text{bits}}{\text{s}\cdot\text{Hz}}\right]$	5.1659	3.228	1.2815
R^* $\left[\frac{\text{bits}}{\text{s}\cdot\text{Hz}}\right]$	5.295	3.550	1.807
ΔR^* %	**12.8**	**32.2**	**52.6**

In the sequel, instrumental numerical results are analysed aiming to corroborate the optimality of the solution.

A. *Throughput vs. Sensing Time*

The behavior of the throughput as a function of the sensing time, i.e. the objective function in (20), can be shown in Fig. 2. For the simulation results, $3 \cdot 10^4$ Monte Carlo simulation (MCS) trials were deployed and compared with the theoretical curve. One can infer that the throughput function has an unique maximum point, witch is the global optimum. Hence, one can conclude that the objective function is concave. Moreover, examining Fig. 2, one can infer by inspection that the maximum value of throughput is achieved in ≈ 2.55 [ms] for the three channel occupancy probability scenarios, in which is confirmed by the solution of the optimization problems.

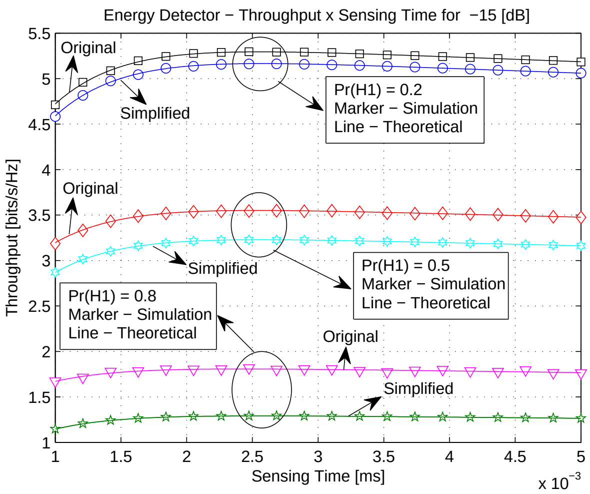

Fig. 2. Throughput *vs.* sensing time for $\mathrm{SNR}_p = -15$[dB].

B. *Probability of Detection vs. Threshold*

In order to obtain the probability of detection *vs.* threshold of the energy detector operating under the optimum sensing time, a number of MCS realizations equal to $3 \cdot 10^4$ trials was chosen. Fig. 3 depicts the probability of detection *vs.* threshold adopting $\tau^* f_s = 15600$ samples. We have compared values in which the channel occupancy probability are low, medium, high and when the channel is completely occupied. Examining Fig. 3 one can conclude that the target probability of detection $\overline{P_d} = 0.9$ is obtained for different values of threshold that can be obtained using the equation (9).

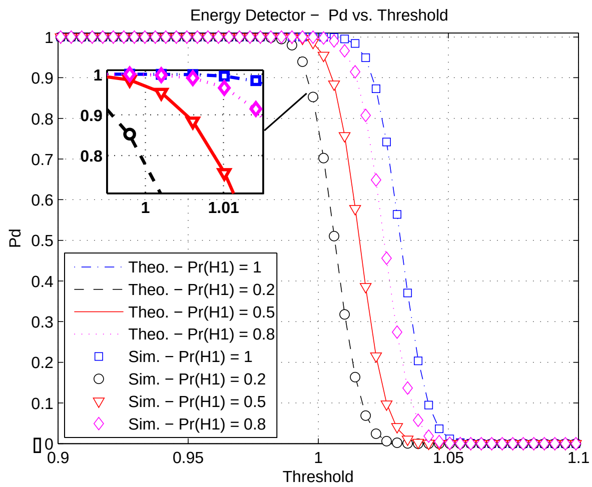

Fig. 3. Probability of detection *vs.* Threshold for $N_s = 15600$ samples.

C. *Probability of Detection vs. Number of Samples*

To obtain the figure of merit described by the probability of detection *vs.* N_s in the context of CRN equipped with ED, the same number of MCS trials ($3 \cdot 10^4$ trials) was chosen. As a consequence, Fig. 4 shows the probability of detection *vs.* N_s adopting value of the $\overline{P_d} = 0.9$ and the PU SNR value is $\mathrm{SNR}_p = -15$ [dB]. Notice that in Fig 4, we compare values in that the probability of the channel is occupied are low, medium, high and when the channel is completely occupied; notice that a guaranteed 0.9 probability of detection is attained under the four scenarios when $N_s \geq 15600$ samples.

V. FINAL REMARKS

An optimization problem was formulated to deal with the sensing *vs.* throughput tradeoff (STO problem) in a CRN with one PU and one SU in a single-band spectrum sensing scheme. The equivalent and simplified optimization problem is convex but nonlinear in τ, and can be solved optimally using efficient solvers, such

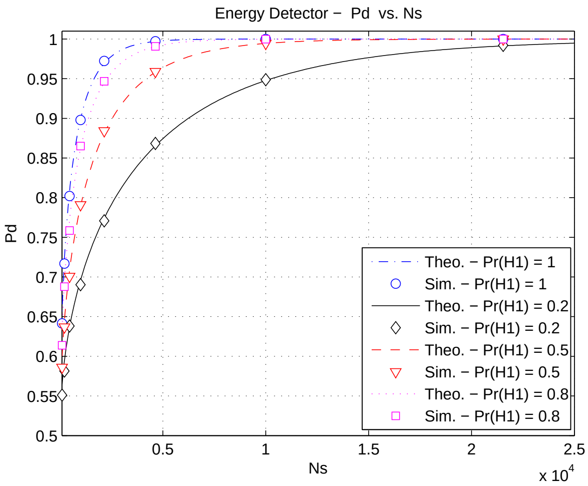

Fig. 4. Probability of Detection *vs.* N_s.

as *fmincon* tool of *MATLAB Optimization Toolbox*. The numerical solutions discussed in this paper confirm that the maximum is a global optimum and the objective function is a concave function.

Comparing the values of P_d with values of threshold one can see that different values of threshold implies in values of $P_d \geq 0.9$, which respects the constraints. Comparing values of P_d with values of N_s and values of sensing time, it is possible to conclude that for values of $N_s \geq 15600$ samples, implies in values of P_d above 0.9, with no violation of the constraints limits. Hence, we concluded that the obtained solution respect the constraint of the optimization problem, in addition to maximize the throughput of the SU.

APPENDIX

PROOF OF CONCAVITY

The convexity of the STO problem (21) is equivalent to demonstrate the concavity of the objective function $\widehat{R}(\tau)$ in (20), since the constraint set is convex, *i.e.* the constraint `(C.1.)` is a affine function in relation to τ and `(C.2.)` is a concave function regarding τ, as predicted by (6) and confirmed by numerical values of Fig. 4.

Clearly, the objective function (20) depends on the false alarm probability function that can be written as

$$\widehat{R}(\tau) = \frac{T-\tau}{T} C_0 \cdot [1 - P_f(\tau)] \cdot \mathrm{P_r}(\mathcal{H}^0). \tag{23}$$

The proof can be done using the concept of composition of convex functions [12] that preserve the concavity of objective function (20). The values of C_0 and $\mathrm{P_r}(\mathcal{H}^0)$ are constants. The function $\frac{T-\tau}{T}$ is an affine function, i.e the function is concave. Then, it is necessary to prove that $[1 - P_f(\tau)]$ is concave.

In order to ensure that $[1 - P_f(\tau)]$ is concave, the false alarm probability function must be convex. Taking the first derivative of the false alarm probability function, eq. (5), we obtain:

$$\frac{dP_f(\tau)}{d\tau} = -\frac{\mathrm{SNR}_p\sqrt{f_s}}{2\sqrt{2\pi\tau}} e^{-0.5\left(\sqrt{2\mathrm{SNR}_p+1}Q^{-1}(\overline{P_d})+\mathrm{SNR}_p\sqrt{\tau f_s}\right)^2}.$$

Hence, assuming $\tau > 0$ implies that $\frac{dP_f(\tau)}{d\tau} < 0$, i.e $P_f(\tau)$ is convex in τ when subject to $P_f(\tau) \leq 0.5$.

Finally, ensuring that $\frac{dP_f(\tau)}{d\tau}$ is negative and increasing in τ, *i.e.* false alarm probability function is convex, it follows that $[1 - P_f(\tau)]$ is concave in τ. Hence, $\widehat{R}(\tau)$ is concave in τ. ∎

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