Achieving the Sum-Rate for Symmetric Gaussian Interference Channels Using LDPC Codes

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***Abstract*— In this paper, we resort to results presented in the Costa's paper [1], and employ LDPC (low-density parity-check) codes to investigate the achievable sum-rates for the Gaussian Interference Channel (GIC). We assume a symmetric GIC and the same power for both users. A different decoding scheme is employed for each interference region.**

For the weak interference region, user's interference is treated as noise. For the moderate interference region, we use *non-naive* time division multiplexing (TDM) approach. For the strong interference region, joint decoding is employed and finally, for the very strong interference region, the SIC (successive interference cancellation) method is used. Numerical simulations have been performed to show that the gap between the Han and Kobayashi limit and the achievable sum-rate is very small.

***Keywords*— Interference Channel, LDPC, Interference regions.**

I. Introduction

There are several issues still unsolved to be addressed in communication systems involving many senders and receivers. One of these classical problems is the Gaussian interference channel (GIC) where two transmitters send information for two receivers sharing the same channel. The use of a common channel naturally causes mutual interference between the users.

The capacity region for the Gaussian interference channel (GIC) is not known in general [2], but important results were proved by Carleial [3]. Later, in 1981, Sato [4] showed striking results about the capacity region of a GIC under strong and very strong interference. Independently, Han and Kobayashi [5] obtained results that contain Sato's results. The Han-Kobayashi's region is not the capacity region for all the interference regimes, but it is the best known achievable rate region. For the particular cases of strong and very strong interference, it is the capacity region and can be computed as the intersection of the regions of two multiple access channels (MACs).

In [6], authors have used the Han-Kobayashi's scheme in order to design low-density parity-check (LDPC) codes. The authors have used techniques such as *EXIT Charts* and were able to achieve very good performance under all interference regimes using joint decoding schemes.

In this work, we resort to the results present in [1], and we show that it is also possible to achieve sum-rate points close to the Han-Kobayashi limit using simple techniques such as time sharing, successive interference cancellation, and for some interference regime, joint decoding. Note that our analysis is restricted to non-optimized LDPC codes and moderate length. We deal with a two-user symmetric GIC, that is, the interference parameter is the same for both users. This channel applies to different scenarios in communication systems such as the wireless communication between base station and mobile units and also in the satellite links.

Our analysis is based on the Costa's paper [1], where four regions were identified: weak, moderate, strong and very strong. For each scenario, we employ an appropriate decoding scheme: single-user, time division multiplexing (TDM), joint decoding and successive interference cancellation (SIC), respectively. In a single-user decoding, only a single decoder is needed, since, in this case, the signal of the second user is treated as noise. In the TDM decoding scheme, users can share the same code transmitting at alternating time slots [7]. Joint decoding is an iterative decoding scheme that employs the concepts of message-passing [8] and factor graphs [9]. Finally, SIC decoding consists of a single-user decoding scheme, applied to cancel out the message that is interpreted as noise, succeeded by another single-user decoding scheme, used to decode the message of interest [10].

The rest of this paper is organized as follows. In Section II, we present the system model and consider encoding and decoding schemes adopted in the simulations. In Section III, we describe the capacity regions of the GIC in the four scenarios of interference. The numerical results together with an analysis of the achievable rate sum are drawn in Section IV. Finally, conclusions are given in Section V.

II. System Description and Problem Statement

We consider a network which involves four users, two senders and two receivers. Each sender is interested in communicating with only one receiver – in other words, there is a one-to-one correspondence between senders and receivers. However, a message from a sender arrives at both receivers. Thus, each sender causes interference into the other's transmission.

More specifically, we study a two-user discrete memoryless GIC. A general scheme for this channel is given in Figure 1. The GIC is defined in terms of two finite alphabet inputs W_1 and W_2. The encoder 1 maps W_1 into $X_1 = (X_{(1)1}, X_{(2)1}, \ldots, X_{(n)1})$ and encoder 2 maps W_2 into $X_2 = (X_{(1)2}, X_{(2)2}, \ldots, X_{(n)2})$. Sources are independent, i.e., there is no cooperation between them. In addition, the input signals must comply with the power restrictions below:

$$\frac{1}{n}\sum_{i=1}^{n} X_{(i)1} \leq P_1 \qquad \frac{1}{n}\sum_{i=1}^{n} X_{(i)2} \leq P_2.$$

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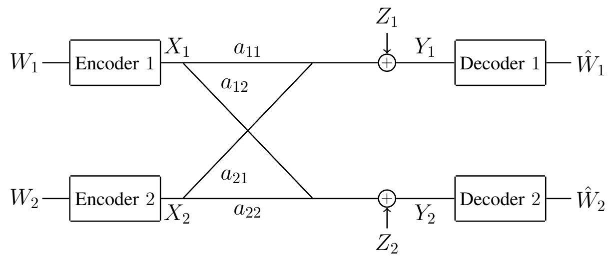

Fig. 1. Scheme for a two-user Gaussian Interference Channel.

Sender 1 chooses a codeword from his codebook and transmits over the channel; sender 2 does the same. At reception, each user Y_j observes its signal of interest added by the signal from the other user and by the noise of the channel. Hence, receivers suffer interference from the channel as well as from the other link that shares the channel.

The above model can be defined by the following equations:

$$Y_1 = a_{11}X_1 + a_{21}X_2 + Z_1,$$
$$Y_2 = a_{12}X_1 + a_{22}X_2 + Z_2,$$

where a_{ij} is the coefficient from user i to receiver j. The noises Z_1 and Z_2 are independent identically distributed zero mean Gaussian random variables $Z_i \sim \mathcal{N}(0, N)$, $i = 1, 2$.

In this paper, we consider the channel in the standard form. As shown by Carleial [11], we can reach this form by appropriately scaling a_{11} and a_{22} to obtain $a_{11} = a_{22} = 1$. We also assume symmetric interference in the channel, i.e., $a_{12} = a_{21} = a$. Finally, transmission powers are equal, namely, $P_1 = P_2 = P$, and noise power is unitary $N = 1$. Then, the channel is totally characterized by the interference parameter a, and power P.

A rate pair (R_1, R_2) is said to be achievable if there exists a sequence of length n of $((2^{nR_1}, 2^{nR_2}), n)$ codes with probability of error $p_e^n \to 0$ as $n \to \infty$ [7]. Moreover, a sum-rate k is said to be achievable if there exists an achievable rate pair (R_1, R_2) such that $R_1 + R_2 \leq k$. The capacity region of a channel is defined as the closure of the sets of all achievable rate pairs.

A. BI-AWGN Capacity

We assume a Binary-Input Additive White Gaussian Noise channel (BI-AWGN), since we are interested in the use of binary LDPC codes. Note that the capacity of the BI-AWGN channel can be approximated by the AWGN capacity for rates below 0.6, as can be seen in [12, Fig. 1].

B. Encoding

LDPC codes are linear and can be characterized for their sparse parity-check matrix, and their decoding is performed using the well known sum-product algorithm (SPA) [13]. A code is said regular when the weight w_r of a row and w_c of a column of the matrix $\mathbf{H_{m\times n}}$, satisfy the following relations: $w_r = w_c(n/m)$ and $w_c << m$ [13]. For the irregular codes, the weight of the rows and columns of the matrix are not constant values. In this paper, we employ irregular LDPC codes since they present superior performance when compared to regular codes. MacKay and Neal [14] presented results of LDPC codes near to Turbo codes performance [15]. There are many works that explore this subject in more detail; see, for example, the work by Urbanke et al.[16].

C. Decoding

According to the interference region, three different decoding schemes will be employed.

1) Single User Decoding: This is the traditional single user user decoding method where the decoder employs a factor graph with the sum-product algorithm [9].

2) SIC - Successive Interference Cancellation: In this scheme, the decoding is performed in two steps. The strong user j is first decoded and its information cancelled out from the received signal. Then, the weak user i is finally decoded, as shown in Figure 2.

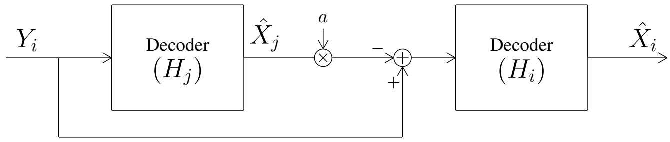

Fig. 2. SIC Decoding Scheme.

3) Joint Decoding: To achieve the results of this work, we adopt different decoding schemes: single user decoding and joint decoding. In contrast to others decoders, a joint decoder employs two separate encoders that perform the decoding of the message simultaneously. This is different, for example, from the SIC, which is performed in two serial stages. Joint decoding is based on factor graphs [17],[18], [19] which connect the users through states-check nodes. These nodes accomplish the exchange of messages between the users through edges of the graph, as can be seen in Figure 3.

An advantage of using a joint decoder is that the decoder is seen as a set of decoders of low complexity which perform local operations. At the decoder, the *log-likelihood* ratio (LLRs) between the individual variable nodes and check nodes is computed as [20], [13]

$$L(v_j|\mathbf{y}) = \log_2\left(\frac{Pr(v_j = 0|\mathbf{y})}{Pr(v_j = 1|\mathbf{y})}\right),$$

where v_j is variable node corresponding to the jth bit of the codeword and $\mathbf{y}$ is the received information. For each iteration, the state check nodes compute the following operation

$$sv_j^{[2]} = \log_2\left(\frac{\mathcal{P}_{00}e^{vs^{[1]}} + \mathcal{P}_{10}}{\mathcal{P}_{01}e^{vs^{[1]}} + \mathcal{P}_{11}}\right), \quad (1)$$

where $\mathcal{P}_{00}$, $\mathcal{P}_{01}$, $\mathcal{P}_{10}$, and $\mathcal{P}_{11}$ are the a posteriori probabilities.

III. INTERFERENCE REGIMES

It is well known that the capacity region of the GIC is well defined for the strong and very strong interference [4], [5], whereas the capacity regions under weak and moderate

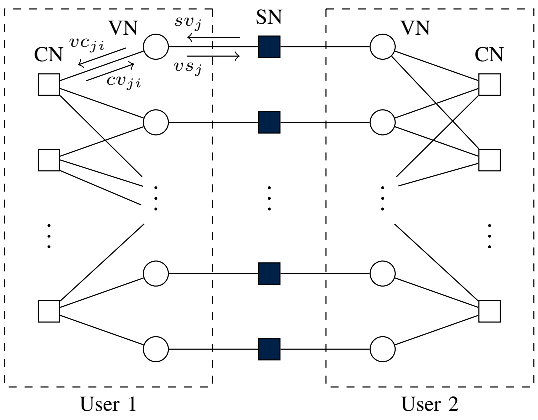

Fig. 3. Two-user MAC factor graph.

interference are yet to be completely established. Following the Costa's paper [1], our analysis is based on the achievable rate sum $R_1 + R_2$, where we have considered a symmetric interference, given by parameter a (non-negative) and power P.

We can divide the regions of interference as a function of the parameter a. For the cases where $a < a^* = \frac{1}{\sqrt{1+\sqrt{2P}}}$, the interference is considered weak. The moderate interference region occurs for $\frac{1}{\sqrt{1+\sqrt{2P}}} \leq a < 1$. The strong interference region occurs for $1 \leq a < \sqrt{1+P}$. Finally, the very strong interference region occurs for values of $a \geq \sqrt{1+P}$ [1].

A. Weak Interference

Although the capacity region is not known in the weak interference regime, the best strategy is to consider the interference as a noise and perform single user decoding method. Therefore, the following achievable rates are attained:

$$R_1 \leq \frac{1}{2}\log_2\left(1 + \frac{P}{1+a^2P}\right), \tag{2}$$

$$R_2 \leq \frac{1}{2}\log_2\left(1 + \frac{P}{1+a^2P}\right), \tag{3}$$

$$R_1 + R_2 \leq \log_2\left(1 + \frac{P}{1+a^2P}\right), \ \text{ if } 0 \leq a < a^*. \tag{4}$$

B. Moderate Interference

Again, although the capacity region is not known in the moderate regime, the best achievable rate for this scheme is attained when the *non-naive* TDM is employed [2]. For this scheme, the rate sum is given by

$$R_1 + R_2 \leq \frac{1}{2}\log_2\left(1+2P\right), \text{ if } a^* \leq a < 1. \tag{5}$$

C. Strong Interference

For this interference regime, the capacity region is known and is given by the intersection of two MACs channels, as exemplified in Figure 4 for $a = 1.1$.

The multiple access channels 1 and 2 are defined as follows: if we consider the transmission of user 1 and user 2 to receiver 1, we can define the first MAC. In the same way, considering user 1 and user 2 transmitting to receiver 2, we can define the second MAC. In Figure 4, the single user capacity is defined as $C_1 = C_2 = \log_2\left(1+P\right)$.

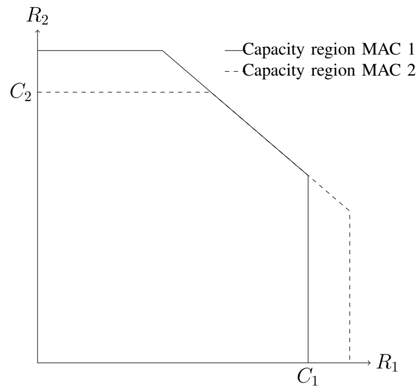

Fig. 4. Intersection of two MACs with $a = 1.1$.

In this regime, the decoding scheme becomes more intricate. To achieve points at the border of the capacity region, there are two options: joint decoding or rate-splitting [21],[22]. In this paper, we have used the joint decoding method as described in Section II-C.3. For this case, the rate sum is given by

$$R_1 + R_2 \leq \frac{1}{2}\log_2\left(1+P+a^2P\right), \text{ if } 1 \leq a < \sqrt{1+P}. \tag{6}$$

Joint decoding schemes for the two-user MAC channel were shown by Roumy e Declercq [20] and by Balatsoukas-Stimming [23].

D. Very Strong Interference

In this regime, the capacity region is also known and given by the intersection of two MACs as illustrated in Figure 5, for $a = 1.7$.

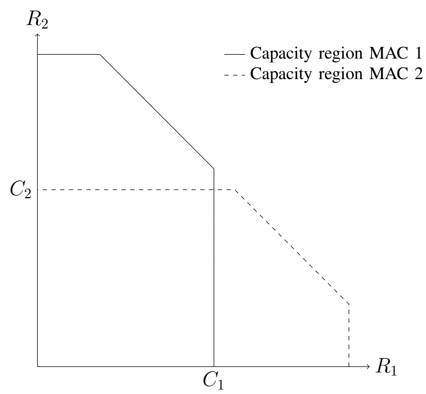

Fig. 5. Intersection of two MACs with $a = 1.7$.

This scheme uses the SIC decoding method, as explained in Section II-C.2. In the first step, the receiver 1 decodes the message of user 2 considering the signal of user 1 as noise. In this case, assuming $N = 1$, we obtain a rate for user 1 equals to $R_1 = \frac{1}{2}\log_2(1 + (P/(1+P))$. Then, at the second step, the decoded message from user 2 is subtracted (or canceled out) from the received signal. Hence, after the channel is cleaned, it remains only the message from user 1 added of noise. Thus, we obtain a rate for the second user equal to $R_2 = \frac{1}{2}\log_2(1+P)$.

In this regime, we have the following achievable rates:

$$R_1 \leq \frac{1}{2}\log_2(1+P), \tag{7}$$

$$R_2 \leq \frac{1}{2}\log_2(1+P), \tag{8}$$

$$R_1 + R_2 \leq \log_2(1+P), \text{ if } a \geq \sqrt{1+P}. \tag{9}$$

The SIC scheme using LDPC codes was shown by Xu and Goertz [24].

IV. Numerical Results

In this section, we present simulations of the transmission over the symmetrical GIC considering four interference regions. The aim of our simulation is to validate the theoretical results presented in the previous sections. In the weak, moderate, and very strong regions, we have used irregular LDPC codes with block length equal to $n = 64800$. In the strong interference region, we have used an irregular LDPC code (MacKay's code) of block length $n = 1000$ employing joint decoding. We consider a transmission to be reliable if it has a bit error rate (BER) of at most 10^{-5}. We arbitrarily set the total power $P = 1.5$, which produces the limiting value between weak and moderate interference as $a^* = 0.57$.

Figure 6 shows the theoretical and some simulated points for the sum rate as a function of the interference parameter a.

A. Weak Interference

In the case of weak interference (where $0 \leq a < a^*$), we have used $R_1 = R_2 = 0.6$, which results in a rate sum of $R_1 + R_2 = 1.2$. Note that the individual rates should obey the rates given in (4) ($R_1 \leq 0.63$, $R_2 \leq 0.63$). In this case, we have obtained a BER near to 10^{-5} within 1 dB to the achievable limit.

B. Moderate Interference

For this region, (5) produces a rate sum as $R_1 + R_2 \leq 1$. The interference parameter used was $a = 0.7$ and the individual rates were $R_1 = R_2 = 0.45$. Again, for this case, we have obtained a BER near to 10^{-5} within 1 dB to the achievable limit.

C. Strong Interference

Using an interference parameter $a = 1.2$, (6) produces a rate sum bound of $R_1 + R_2 \leq 1.05$. In this case, we have used a joint decoding method with $n = 1000$, and two codes with rates $R_1 = 0.5$ and $R_2 = 0.5$. For this case, we have obtained a BER near to 10^{-5} within 3 dB to the sum rate limit. The increase in the gap arises due to the small block length n, as compared to the other cases. Unfortunately, the computational complexity for the joint decoding is prohibitive and requires a small n.

D. Very Strong Interference

In order to simulate the very strong interference region, we have used an interference parameter $a = 1.7$. In this case, (9) results in $R_1 \leq 0.66$, $R_2 \leq 0.66$, and $R_1 + R_2 \leq 1.32$. For this case, the SIC decoding method produces a BER of 10^{-5} within 0.9 dB to the sum rate limit.

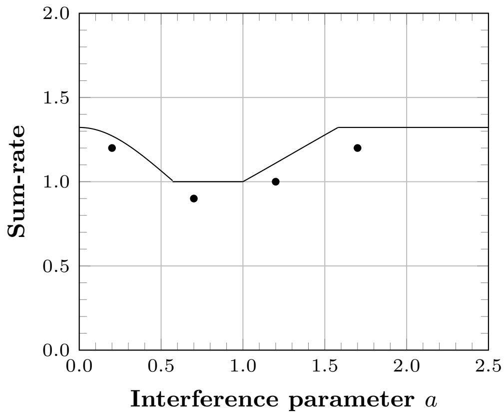

Fig. 6. Achieved Sum-rate for $P = 1.5$.

V. Conclusion

In this paper, we have investigated practical operating points near to the best known achievable rates for the symmetric Gaussian interference channel. We have presented four types of decoding using irregular low-density parity-check codes. The strong regime has demanded the most intricate decoding method, the joint decoding based on factor graphs. Our notable contribution in this work is achieving rate pairs close to the boundaries employing implementable and practical channel codes using simple techniques for all regions of interference.

References

[1] M. H. M. Costa, "On the gaussian interference channel," *IEEE Trans. Inf. Theory*, vol. 31, no. 5, pp. 607–615, Sep 1985.

[2] G. Kramer, *Topics in Multi-User Information Theory*, ser. Foundations and trends in communications and information theory. Now Publishers, 2008. [Online]. Available: http://books.google.com.br/books?id=b7RzVR3mDHkC

[3] A. Carleial, "Interference channels," *IEEE Trans. Inf. Theory*, vol. 24, no. 1, pp. 60–70, Jan. 1978.

[4] H. Sato, "The capacity of the gaussian interference channel under strong interference (corresp.)," *IEEE Trans. Inf. Theory*, vol. 27, no. 6, pp. 786–788, Nov. 1981.

[5] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49–60, 1981.

[6] S. Sharifi, A. K. Tanc, and T. M. Duman, "On ldpc codes for gaussian interference channels," in *Information Theory (ISIT), 2014 IEEE International Symposium on*, June 2014, pp. 1992–1996.

[7] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley, 2006. [Online]. Available: http://books.google.com.br/books?id=EuhBluW31hsC

[8] T. J. Richardson and R. L. Urbanke, “The capacity of low-density parity-check codes under message-passing decoding,” *IEEE Trans. Inf. Theory*, vol. 47, no. 2, pp. 599–618, Feb. 2001.
[9] F. R. Kschischang, B. J. Frey, and H.-A. Loeliger, “Factor graphs and the sum-product algorithm,” *IEEE Trans. Inf. Theory*, vol. 47, no. 2, pp. 498–519, 2001.
[10] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, ser. Wiley series in telecommunications. Cambridge University Press, 2005. [Online]. Available: http://books.google.com.br/books?id=66XBb5tZX6EC
[11] A. Carleial, “A case where interference does not reduce capacity (corresp.),” *IEEE Trans. Inf. Theory*, vol. 21, no. 5, pp. 569–570, Sep. 1975.
[12] J. G. David Forney and G. Ungerboeck, “Modulation and coding for linear gaussian channels,” *IEEE Trans. Information Theory*, vol. 44, no. 6, pp. 2384–2415, Oct. 1998.
[13] W. Ryan and S. Lin, *Channel Codes: Classical and Modern*. Cambridge University Press, 2009. [Online]. Available: http://books.google.com.br/books?id=0gwqxBU_t-QC
[14] D. J. C. MacKay and R. M. Neal, “Near shannon limit performance of low density parity check codes,” *Electron. Lett.*, vol. 32, no. 18, pp. 1645–, 1996.
[15] C. Berrou and A. Glavieux, “Near optimum error correcting coding and decoding: turbo-codes,” *IEEE Trans. Commun.*, vol. 44, no. 10, pp. 1261–1271, 1996.
[16] T. J. Richardson and R. L. Urbanke, “Efficient encoding of low-density parity-check codes,” *IEEE Trans. Inf. Theory*, vol. 47, no. 2, pp. 638–656, 2001.
[17] J. Boutros and G. Caire, “Iterative multiuser joint decoding: unified framework and asymptotic analysis,” *IEEE Trans. Inf. Theory*, vol. 48, no. 7, pp. 1772–1793, Jul. 2002.
[18] M. A. Khojastepour, N. Ahmed, and B. Aazhang, “Code design for the relay channel and factor graph decoding,” in *Signals, Systems and Computers, 2004. Conference Record of the Thirty-Eighth Asilomar Conference on*, vol. 2, Nov. 2004, pp. 2000–2004.
[19] A. P. Worthen and W. E. Stark, “Unified design of iterative receivers using factor graphs,” *IEEE Trans. Inf. Theory*, vol. 47, no. 2, pp. 843–849, Feb. 2001.
[20] A. Roumy and D. Declercq, “Characterization and optimization of ldpc codes for the 2-user gaussian multiple access channel,” *EURASIP Journal on Wireless Communications and Networking*, vol. 2007, no. 1, p. 074890, 2007. [Online]. Available: http://jwcn.eurasipjournals.com/content/2007/1/074890
[21] B. Rimoldi and R. Urbanke, “A rate-splitting approach to the gaussian multiple-access channel,” *IEEE Trans. Inf. Theory*, vol. 42, no. 2, pp. 364–375, Mar. 1996.
[22] A. J. Grant, B. Rimoldi, R. L. Urbanke, and P. A. Whiting, “Rate-splitting multiple access for discrete memoryless channels,” *IEEE Trans. Inf. Theory*, vol. 47, no. 3, pp. 873–890, Mar. 2001.
[23] A.-K. Balatsoukas-Stimming, “Design of ldpc codes for the two-user gaussian multiple access channel,” Ph.D. dissertation, Technical University of Crete, 2012.
[24] X. Xu and N. Goertz, “Practical successive interference cancellation in the binary-input gaussian multiple-access channel,” in *Source and Channel Coding (SCC), 2008 7th International ITG Conference on*, Jan. 2008, pp. 1–6.