Combining Relay and Direct Links for Joint Symbol and Channel Estimation in MIMO Amplify-and-Forward (AF) Systems

Leandro Ronchini Ximenes and André Lima Ferrer de Almeida

***Abstract*— The non-regenerative Amplify-and-Forward (AF) relaying is a powerful yet simple way to extend signal coverage in wireless communications. Without any decoding procedure, relay stations operating under this protocol offers a very small cost of deployment and operation in comparison with regenerative protocols. However, the AF protocol has its disadvantages, particularly the challenge of estimating the channel matrices that compose the relay link. Besides, few works were dedicated to estimate those channels without using bandwidth-consuming pilot symbols. We resort to a recent nested PARAFAC tensor modeling of the considered relay MIMO system, and then we propose a semi-blind receiver that combines the signals from the relay link with an eventual direct link, providing an improved symbol estimation along with the identification of the channels that compose the relaying network. Compared with an inspiring previous receiver that exploits the PARATUCK2 tensor model, our new receiver have presented consistent coding gains.**

***Keywords*— Amplify-and-forward (AF), tensor decomposition, nested PARAFAC, semi-blind receiver, channel estimation, relay**

I. INTRODUCTION

The deployment of relay stations between communicating terminals, working to enhance the signal power and diversity at the signal destination node, has become a major interest in the contemporary wireless communications [1], [2]. Among the different types of relaying protocol, the Amplify-and-Forward (AF) technique has a deserved appeal due to its simplicity. In this protocol the relay performs the most basic of operations, simply forwarding the signals received from the source node to a destination one with increased power [3]. However, further progressing in the use of cooperative AF relaying also brought new challenges. Due to the fact that in this protocol the relay abstains itself from a costly decoding process, conventional techniques of channel estimation are enable to dissociate the channel coefficients of the source-relay and relay-destination links. While the channel state information (CSI) of each link is not always necessary for symbol estimation, since some classic receivers such as Zero-Forcing (ZF) requires only the knowledge of the global channel between source and destination, it is still crucial for a number of optimization techniques [4], [5].

To solve this estimation issue in MIMO AF relay systems, a promising approach resorts to tensor decompositions. Through different strategies of coding at the source and relay, the signals received at the destination node could be organized into multidimensional arrays, where each mode (dimension) would correspond to one of several possible diversities to be explored (e.g. space, code, time). One of the characteristics of the tensor decompositions, apart from using it as powerful analytical tools, is that the uniqueness of their factors may be guaranteed by more relaxed conditions than matrix-based approaches.

Resorting to the different tensor decompositions proposed in the last 20 years, tensor-based works for wireless MIMO communications have been roughly segmented into two categories: channel estimation in relaying communications using pilot symbols [6], [7], and blind estimation of symbols and channels in point-to-point (P2P) systems [8], [9]. While the works in the first group estimate the channels in a relay system, by sacrificing part of the bandwidth to transmit training (pilot) symbols, the other works deploy efficient blind symbol estimation procedures, but are unable to estimate channels in relaying scenarios.

Only recently the (semi-)blind estimation in relay AF MIMO systems has been proposed in the literature [10]–[13]. These works give one step ahead into the combination of the main aspects of the tensor-based works cited in the previous paragraph. They employ different AF relay strategies, which are needed for individual channel estimation, with a tensor-based coding at source, which allows blind symbol estimation. They use a Khatri-Rao space-time (KRST) coding [14] at the source, so the signals arriving at the receiver via relay link can be either modeled by a PARATUCK2 decomposition [15] or by a nested PARAFAC decomposition [16]. In both cases, signals coming via direct link follows a PARAFAC decomposition [17].

Therefore, due to the KRST coding at the source, symbols and channels can be jointly estimated via any available link – usually the strongest of them. However, it is natural that both links be exploited to increase spatial diversity. The direct link is particularly easy to be exploited in conjunction with such relaying schemes because their signals arrive at the destination in distinct moments (hops), since the signals from the relay are delayed due to its intrinsic forwarding transmission. This is the main concept of the Combined PARAFAC/PARATUCK2 (CPP-ALS) receiver proposed in [10]. More than just treating the signals arriving from the direct link in an independent, separated process, th CPP-ALS exploits the direct link model internally in their estimation framework, improving not only

Leandro R. Ximenes, Faculty of Technology (FT), State University of Campinas (UNICAMP), Limeira-SP, Brazil, E-mail: leandro@ft.unicamp.br. André L. F. de Almeida, Wireless Telecommunication Research Group (GTEL), Federal University of Ceará (UFC), Fortaleza-CE, Brazil, E-mail: andre@gtel.ufc.br

symbol estimation, but refining the channel estimates.

Using the CPP-ALS receiver as baseline, this work proposes a combined semi-blind receiver based on the nested PARAFAC relay modeling. Hence, not only relay and direct links can be used to improve joint symbol and channel estimation, but also this model has structural characteristics that leads to more flexible identifiability conditions and cheaper complexity costs. Bit Error Rate (BER) simulations corroborate the coding gains of the proposed receiver over CPP-ALS.

Notations: Scalars, column vectors, matrices, and tensors are denoted by lower-case (x), boldface lower-case $(\mathbf{x})$, boldface capital $(\mathbf{X})$, and calligraphic $(\mathcal{X})$ letters, respectively. $\mathbf{X}^T$, $\mathbf{X}^*$, $\mathbf{X}^\dagger$, $\mathbf{X}_{l\cdot}$ and $\mathbf{X}_{\cdot m}$ are the transpose, the conjugate, the pseudoinverse, the l^{th} row, and the m^{th} column of $\mathbf{X} \in \mathbb{C}^{L\times M}$, respectively. $D_n(\mathbf{X})$ stands for the diagonal matrix formed from the elements of $\mathbf{X}_{n\cdot}$. Given a third-order tensor $\mathcal{X} \in \mathbb{C}^{I\times J\times K}$, with entry $x_{i,j,k}$, the matrices $\mathbf{X}_{JK\times I}$, $\mathbf{X}_{KI\times J}$ and $\mathbf{X}_{IJ\times K}$ denote tall mode-1, mode-2 and mode-3 unfoldings, with $x_{i,j,k} = [\mathbf{X}_{JK\times I}]_{(k-1)J+j,i} = [\mathbf{X}_{KI\times J}]_{(i-1)K+k,j} = [\mathbf{X}_{IJ\times K}]_{(j-1)I+i,k}$.

A PARAFAC decomposition [17] of a third-order tensor $\mathcal{X} \in \mathbb{C}^{I\times J\times K}$, with matrix factors $(\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3)$, may be noted $\|\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3\|$. Tall and flat mode-1 matrix unfoldings of $\mathcal{X}$ are respectively given by

$$\mathbf{X}_{JK\times I} = (\mathbf{A}_3 \diamond \mathbf{A}_2)\mathbf{A}_1^T, \tag{1}$$

$$\mathbf{X}_{I\times JK} = \mathbf{A}_1(\mathbf{A}_3 \diamond \mathbf{A}_2)^T = (\mathbf{X}_{JK\times I})^T \tag{2}$$

where $\diamond$ denotes the Khatri-Rao product.
Similar mode-2 and mode-3 unfoldings can be obtained by permuting the factor matrices, i.e. respectively

$$\mathbf{X}_{KI\times J} = (\mathbf{A}_1 \diamond \mathbf{A}_3)\mathbf{A}_2^T = (\mathbf{X}_{J\times IK})^T, \tag{3}$$

$$\mathbf{X}_{IJ\times K} = (\mathbf{A}_2 \diamond \mathbf{A}_1)\mathbf{A}_3^T = (\mathbf{X}_{K\times IJ})^T. \tag{4}$$

II. System model

We consider a one-way relay MIMO system, where the communication is divided into two hops (Fig. 1). During the first one, the source node transmits the signals to the relay (*SR* link) and via any existing direct link to the destination (*SD* link). In the second hop the source stays silent, while the relay forwards amplified signals to the destination node (*RD* link). The cascading of the *SR* and *RD* links is termed *SRD* link. M_D, M_S and M_R are the number of antennas at destination, source and relay nodes, respectively. The communication channels are considered flat-fading and invariant during the transmission protocol.

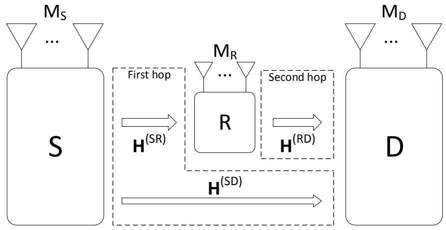

Fig. 1. One-way model.

Let $\mathbf{S} \in \mathbb{C}^{N\times M_S}$ be a matrix containing N data-streams of M_S symbols multiplexed to the M_S source antennas. A simplified Khatri-Rao space-time (KRST) coding [14] is carried out by introducing a temporal redundancy with the code $\mathbf{C} \in \mathbb{C}^{P\times M_S}$, such that the transmitted signals follow

$$\widetilde{\mathbf{S}}_{M_S\times NP} = (\mathbf{C} \diamond \mathbf{S})^T, \tag{5}$$

where P is the source (spreading) code length. Therefore, the transmission by the source consists of sending a block of NP vectors of M_S coded symbols.

The transmitted signals reaching the destination through the direct link channel $\mathbf{H}^{(SD)} \in \mathbb{C}^{M_D\times M_S}$ are given by

$$\begin{aligned}\mathbf{X}^{(SD)}_{M_D\times NP} &= \mathbf{H}^{(SD)}\widetilde{\mathbf{S}}_{M_S\times NP} \\ &= \mathbf{H}^{(SD)}\left(\mathbf{C} \diamond \mathbf{S}\right)^T,\end{aligned} \tag{6}$$

and (6) is a mode-1 unfolding of the receive signal tensor $\mathcal{X}^{(SD)} \in \mathbb{C}^{M_D\times N\times P}$. Keeping the correspondence $(\mathcal{X}, \mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3) \Longleftrightarrow (\mathcal{X}^{(SD)}, \mathbf{H}^{(SD)}, \mathbf{S}, \mathbf{C})$ between (2) and (6), then the mode-2 and mode-3 unfoldings of $\mathcal{X}^{(SD)}$ are given, respectively, by

$$\mathbf{X}^{(SD)}_{PM_D\times N} = (\mathbf{H}^{(SD)} \diamond \mathbf{C})\mathbf{S}^T \tag{7}$$

and

$$\mathbf{X}^{(SD)}_{M_DN\times P} = \left(\mathbf{S} \diamond \mathbf{H}^{(SD)}\right)\mathbf{C}^T. \tag{8}$$

The element $x^{(SD)}_{m_D,n,p}$ of $\mathcal{X}^{(SD)}$ contains the signal collected by the m_D^{th} antenna at the p^{th} repetition of the n^{th} coded data-stream. Assuming that the PARAFAC decomposition of $\mathcal{X}^{(SD)}$ admits an (essential) unique solution [18], the unfoldings (6), (7) and (8) can be used to jointly estimate $\mathbf{H}^{(SD)}$ and $\mathbf{S}$. There are few ways to solve this problem [10], [11], [14], but perhaps the most efficient form is to apply the PARAFAC-SVD algorithm [11], [12]. Although symbol estimates can be fully obtained via direct link using PARAFAC-SVD, in many cases the average Signal-to-Noise Ration (SNR) in this link is low, justifying the addition of an intermediate relay station.

Still after the first hop, the signals received by the relay after transmission through the *source-relay* channel $\mathbf{H}^{(SR)} \in \mathbb{C}^{M_R\times M_S}$ are given by

$$\begin{aligned}\mathbf{W}_{M_R\times NP} &= \mathbf{H}^{(SR)}\widetilde{\mathbf{S}}_{M_S\times NP}, \\ &= \mathbf{H}^{(SR)}\left(\mathbf{C} \diamond \mathbf{S}\right)^T.\end{aligned} \tag{9}$$

At the relay, a new KRST coding is performed on the incoming signals, and forwarded to the destination node through the *relay-destination* channel $\mathbf{H}^{(RD)} \in \mathbb{C}^{M_D\times M_R}$. As shown in [11], the signals at the destination can be written as:

$$\mathbf{X}^{(SRD)}_{JM_D\times NP} = \left(\mathbf{G} \diamond \mathbf{H}^{(RD)}\right)\mathbf{W}_{M_R\times NP}, \tag{10}$$

where $\mathbf{G} \in \mathbb{C}^{J\times M_R}$ is the relay code matrix, and J is its code length. Note that the relaying protocol is non-regenerative, since no decoding is performed at the relay. Code $\mathbf{G}$ is analogous to $\mathbf{C}$, but provides an independent time-spreading.

Replacing (9) in (10) leads to

$$\mathbf{X}^{(SRD)}_{JM_D\times NP} = \left(\mathbf{G} \diamond \mathbf{H}^{(RD)}\right)\mathbf{H}^{(SR)}\left(\mathbf{C} \diamond \mathbf{S}\right)^T. \tag{11}$$

Equation (11) is an unfolding of the fourth-order tensor $\mathcal{X}^{(SRD)} \in \mathbb{C}^{M_D \times J \times N \times P}$ which follows a nested PARAFAC decomposition [11], [16], noticeable by the presence of the two Khatri-Rao products instead of the usual single one in the PARAFAC model unfoldings.

Defining the equivalent *SRD* channel as

$$\mathbf{Z}_{JM_D \times M_S} = \left(\mathbf{G} \diamond \mathbf{H}^{(RD)}\right) \mathbf{H}^{(SR)}, \tag{12}$$

(11) can be rewritten as

$$\begin{aligned} \mathbf{X}^{(SRD)}_{JM_D \times NP} &= \mathbf{Z}_{JM_D \times M_S} \left(\mathbf{C} \diamond \mathbf{S}\right)^T \\ &= \mathbf{Z}_{JM_D \times M_S} \widetilde{\mathbf{S}}_{M_S \times NP}, \end{aligned} \tag{13}$$

where $\widetilde{\mathbf{S}}_{M_S \times NP}$ was defined in (5), and $\mathbf{Z}_{JM_D \times M_S}$ corresponds to a mode-3 unfolding (cf. (4)) of the tensor $\mathcal{Z} \in \mathbb{C}^{M_D \times J \times M_S}$ admitting a PARAFAC model with matrix factors $(\mathbf{G}, \mathbf{H}^{(RD)}, (\mathbf{H}^{(SR)})^T)$. This third-order tensor $\mathcal{Z}$ can be seen as the effective channel that encapsulates channels and the relay code $\mathbf{G}$ matrix.

III. Semi-blind estimation

In [11] the so-called DALS receiver estimates symbol and channel matrices through Alternating Least Squares (ALS) iterative minimizations based on the unfoldings of tensors $\mathcal{X}^{(SRD)}$ and $\mathcal{Z}$. More precisely, $\mathbf{Z}_{JM_D \times M_S}$ and $\mathbf{S}$ are estimated with the labeled ALS-X algorithm by exploiting respectively the matrix unfoldings

$$\mathbf{X}^{(SRD)}_{NP \times M_D J} = \left(\mathbf{C} \diamond \mathbf{S}\right) \mathbf{Z}_{M_S \times JM_D}, \tag{14}$$

$$\mathbf{X}^{(SRD)}_{PJM_D \times N} = \left(\mathbf{Z}_{JM_D \times M_S} \diamond \mathbf{C}\right) \mathbf{S}^T. \tag{15}$$

and then the individual channels $\mathbf{H}^{(SR)}$ and $\mathbf{H}^{(RD)}$ can be estimated by the iterative ALS-Z algorithm, described in Alg. 1. Note that the ALS-Z algorithm exploits the matrix unfoldings of $\widehat{\mathcal{Z}}$, previously estimated by ALS-X. Since $\mathbf{G}$ is known, those unfoldings are notably (12) and the mode-1 unfolding

$$\mathbf{Z}_{JM_S \times M_D} = \left((\mathbf{H}^{(SR)})^T \diamond \mathbf{G}\right) \left(\mathbf{H}^{(RD)}\right)^T. \tag{16}$$

ALS-like algorithms such as ALS-Z estimate one matrix factor at a time, using the other previously estimated factors to minimize its respective cost function.

A. Using direct link: the CALS-X algorithm

Unlike DALS, in the *Sequential PARAFAC/PARATUCK2* (SPP-ALS) and *Combined PARAFAC/PARATUCK2* (CPP-ALS) receivers [10], the direct link can also be combined with the relay link to provide an estimation improvement over the relay link only PT2-ALS receiver. While SPP-ALS simply adopts a (rough) symbol estimation from the direct link to initialize its main algorithm, the CPP-ALS receiver combines the signals from the two links to enhance not only symbol, but also channel estimation. In contrast to DALS described previously, symbols and individual channels with PARATUCK2-based receivers are estimated within a single, heavier iterative process.

Algorithm 1 ALS-Z [11]

- *Estimates partial channels from the channel tensor* $\widehat{\mathcal{Z}}$

Input $\widehat{\mathcal{Z}}$ and $\mathbf{G}$

1: i=0: Initialize $\widehat{\mathbf{H}}^{(RD)}_0$
2: $i \leftarrow i + 1$

$$\widehat{\mathbf{H}}^{(SR)}_i = \left(\mathbf{G} \diamond \widehat{\mathbf{H}}^{(RD)}_{i-1}\right)^\dagger \widehat{\mathbf{Z}}_{M_D J \times M_S}$$

$$(\widehat{\mathbf{H}}^{(RD)}_i)^T = \left((\widehat{\mathbf{H}}^{(SR)}_i)^T \diamond \mathbf{G}\right)^\dagger \widehat{\mathbf{Z}}_{JM_S \times M_D}$$

3: Go to step 2 until convergence.
4: (Optional) eliminate eventual scaling ambiguities.

Output $\widehat{\mathbf{H}}^{(SR)}$ and $\widehat{\mathbf{H}}^{(RD)}$

The proposed CALS-X algorithm focuses on the advantages of the different receivers mentioned in the previous paragraph. It combines relay and direct links to achieve a better estimation performance; and at the same time, its estimation is performed through two sequential iterative algorithms, one being dedicated only to channel estimation.

The code matrices $\mathbf{C}$ and $\mathbf{G}$ are assumed known by receiver. As with the DALS receiver, $\mathbf{H}^{(SR)}$ and $\mathbf{H}^{(RD)}$ can be estimated by using the ALS-Z algorithm (Alg. 1) if they are necessary. Unlike DALS, the proposed CALS-X algorithm estimates $\mathbf{S}$ and $\mathcal{Z}$ by using the estimates of $\mathbf{H}^{(SD)}$ from PARAFAC-SVD. Let us define a combined channel matrix

$$\mathbf{Z}^{(c)} = \begin{bmatrix} \mathbf{H}^{(SD)} \\ \mathbf{Z}_{JM_D \times M_S} \end{bmatrix} \in \mathbb{C}^{K \times M_S} \tag{17}$$

where $K = (M_D + JM_D)$ can be seen as the overall virtual number of antennas for the sake of receive diversity. From (6) and (13), the vertical concatenation of the signals from the direct link and relay link leads to

$$\begin{aligned} \mathbf{X}^{(c)}_{K \times PN} &\triangleq \begin{bmatrix} \mathbf{X}^{(SD)}_{M_D \times NP} \\ \mathbf{X}^{(SRD)}_{JM_D \times NP} \end{bmatrix} \in \mathbb{C}^{K_1 \times NP} \\ &= \mathbf{Z}^{(c)} \left(\mathbf{C} \diamond \mathbf{S}\right)^T. \end{aligned} \tag{18}$$

In similar way, the concatenation of (7) and (15) yields

$$\mathbf{X}^{(c)}_{PK \times N} \triangleq \begin{bmatrix} \mathbf{X}^{(SD)}_{PM_D \times N} \\ \mathbf{X}^{(SRD)}_{PJM_D \times N} \end{bmatrix} \tag{19}$$

$$\begin{aligned} &= \begin{bmatrix} \mathbf{H}^{(SD)} \diamond \mathbf{C} \\ \mathbf{Z}_{JM_D \times M_S} \diamond \mathbf{C} \end{bmatrix} \mathbf{S}^T, \\ &= \left(\mathbf{Z}^{(c)} \diamond \mathbf{C}\right) \mathbf{S}^T, \end{aligned} \tag{20}$$

where we have used the property $\begin{bmatrix} \mathbf{A} \diamond \mathbf{C} \\ \mathbf{B} \diamond \mathbf{C} \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \end{bmatrix} \diamond \mathbf{C}$.

One can see (18) and (20) as respectively mode-1 and mode-2 unfoldings of a third-order tensor $\mathcal{X}^{(c)} \in \mathbb{C}^{K \times N \times P}$ which admits a PARAFAC decomposition with matrix factors ($\mathbf{Z}^{(c)}$, $\mathbf{S}$, $\mathbf{C}$). Therefore, due to similarity between the PARAFAC and nested PARAFAC models, signals from the direct and relay links can be combined in a way that the resulting signal tensor $\mathcal{X}^{(c)}$ also follows a PARAFAC model.

Exploiting the extended unfolded representations in Eqs. (18) and (20), the CALS-X algorithm consists in minimizing in the Least Squares (LS) sense the reconstruction error $e = \|\mathcal{Y}^{(c)} - \widehat{\mathcal{X}}^{(c)}\|_F^2$, where $\mathcal{Y}^{(c)}$ is the (noisy) observation of $\mathcal{X}^{(c)}$. An estimate of $\mathbf{Z}_i^{(c)}$ at the i^{th} iteration can be found by minimizing the following cost function:

$$J(\mathbf{Z}^{(c)}) = \left\| \mathbf{Y}^{(c)}_{PN\times K} - \left(\mathbf{C} \diamond \widehat{\mathbf{S}}_{i-1}\right)\left(\mathbf{Z}^{(c)}\right)^T \right\|_F^2, \quad (21)$$

which gives

$$\left(\widehat{\mathbf{Z}}_i^{(c)}\right)^T = \left(\mathbf{C} \diamond \widehat{\mathbf{S}}_{i-1}\right)^\dagger \mathbf{Y}^{(c)}_{PN\times K_1}. \quad (22)$$

Using $\widehat{\mathbf{Z}}_i^{(c)}$, an estimate of $\mathbf{S}_i$ is then obtained by minimizing the cost function

$$J(\mathbf{S}) = \left\| \mathbf{Y}^{(c)}_{PK\times N} - \left(\widehat{\mathbf{Z}}_i^{(c)} \diamond \mathbf{C}\right)\mathbf{S}^T \right\|_F^2, \quad (23)$$

yielding

$$\widehat{\mathbf{S}}_i^T = \left(\widehat{\mathbf{Z}}_i^{(c)} \diamond \mathbf{C}\right)^\dagger \mathbf{Y}^{(c)}_{PK\times N}. \quad (24)$$

Algorithm 2 CALS-X

- *Jointly estimates symbol and channels by exploiting both relay and direct links*

Input $\mathbf{Y}^{(SRD)}_{NJM_D\times P}$ and $\mathbf{C}$

1: i=0: Initialize $\widehat{\mathbf{S}}_0$ from PARAFAC-SVD.
2: $i \leftarrow i+1$
3: Estimate the combined channel $\mathbf{Z}^{(c)}$ using (22):

$$\widehat{\mathbf{Z}}_i^{(c)} = \left(\mathbf{C} \diamond \widehat{\mathbf{S}}_{i-1}\right)^\dagger \mathbf{Y}^{(c)}_{PN\times K}$$

4: Estimate the symbol matrix $\mathbf{S}$ using (24):

$$\widehat{\mathbf{S}}_i^T = \left(\widehat{\mathbf{Z}}_i^{(c)} \diamond \mathbf{C}\right)^\dagger \mathbf{Y}^{(c)}_{PK\times N}$$

5: Go to step 2 until convergence.
6: Eliminate scaling ambiguities with (25)
7: Extract $\widehat{\mathbf{H}}^{(SD)}$ and $\widehat{\mathbf{Z}}_{JM_D\times M_S}$ from $\widehat{\mathbf{Z}}^{(c)}$.

Output $\widehat{\mathbf{S}}$, $\widehat{\mathbf{H}}^{(SD)}$ and $\widehat{\mathbf{Z}}^{(SRD)}_{JM_D\times M_S}$

A proper estimation of symbols and channels is ensured if **identifiability** and **uniqueness** conditions are satisfied. For the former, from [11] one knows that $\mathbf{C}$ being full column rank is a sufficient condition for DALS, and consequently also for CALS-X, since it ensures that the Khatri-Rao products in (21) and (23) are left-invertible. Likewise, the identifiability of channels matrices holds if $\mathbf{G}$ is full column rank, i.e. $J \geq M_R$. In terms of uniqueness conditions, we work with the following assumptions:

- $\mathbf{S}$ is full column rank, i.e. $N \geq M_S$;
- $\mathbf{H}^{(RD)}$, $\mathbf{H}^{(RD)}$ and $\mathbf{H}^{(RD)}$ are rich-scattering channels;
- $\{M_D, M_R, M_S\} \geq 2$;
- $M_D \geq M_R - M_S + 2$.

In this case, the column scaling ambiguities in the solution of CALS-X can be solved by

$$\widehat{\mathbf{S}} \leftarrow \widehat{\mathbf{S}}\mathbf{\Lambda}^{(S)}, \quad \widehat{\mathbf{Z}}^{(c)} \leftarrow \widehat{\mathbf{Z}}^{(c)}(\mathbf{\Lambda}^{(S)})^{-1}, \quad (25)$$

where $\mathbf{\Lambda}^{(S)} = D_1(\mathbf{S})D_1^{-1}(\widehat{\mathbf{S}})$. Thus, knowing the first row of $\mathbf{S}$ is enough to eliminate the scaling ambiguity matrix $\mathbf{\Lambda}^{(S)}$.

IV. SIMULATIONS

In this section, the BER performance of the proposed CALS-X receiver is evaluated using 10^5 runs of Monte Carlo simulations.The evaluation of the channel estimation performance is not performed in this paper, since from [11] it is known that the effective channel estimation accuracy is directly related to the accuracy of symbol estimation. However, we recall that the channel estimation is performed by following the steps described in Alg. 1 for both DALS and CALS-X.

The source and relay coding matrices $\mathbf{C}$ and $\mathbf{G}$ are (truncated) DFT matrices, and we assume $\mathbf{H}^{(SR)} \sim \mathcal{CN}(0, 1/M_S)$ and $\mathbf{H}^{(RD)} \sim \mathcal{CN}(0, 1/M_R)$. The symbol matrix is expressed as $\mathbf{S} = \sqrt{E_S}\mathbf{S}_o$, where $\mathbf{S}_o$ is composed by unit-power symbols that are randomly drawn from a 8-PSK alphabet at each run, and E_S denotes the symbol energy. The first row of $\mathbf{S}$ is composed of ones to avoid scaling ambiguities with (25). The additive noise samples at relay and destination are complex standard normal random variables. Symbol and channel initializations for DALS are drawn from this same statistical distribution.

For the direct link channel $\mathbf{H}^{(SD)} \sim \mathcal{CN}(0, (1/\alpha)/M_S)$, where α is then the energy difference between the relay and direct links. Changing J, N or P does not affect *a priori* the average SNR of any link at destination, since both energies of the noise-free signals and of the addictive noises are proportional to these parameters. In the following simulations, α is given in dB, where $\alpha = 0$ dB means a very strong link and $\alpha = 20$ dB a very poor one compared to the relay link..

A. CALS-X versus PT2-ALS/CPP-ALS [10] versus DALS [11]

The use of the direct link in the CALS-X algorithm is important for two purposes: for initialization and to provide additional spatial diversity for the symbol estimation. In spite of the fact that the initialization of CALS-X with symbols estimated from PARAFAC-SVD means *a priori* an additional task for the receiver, a counterpoint is the expected reduction of the number of iterations of CALS-X. In addition, the fast PARAFAC-SVD would be finalized before the end of the second hop and of the start of CALS-X, thus does not imposing a processing bottleneck.

In addition, the use of the direct link is supposed to bring an improvement in symbol and channel estimation, mirroring the performance gains obtained by the CPP-ALS receiver [10]. In Fig. 2 the performance of the CALS-X algorithm is compared to those of PT2-ALS and DALS (both without direct link) and also to CPP-ALS (both with direct link). A strong link ($\alpha = 0$ dB) is used. Compared with the DALS algorithm, this figure shows how the CALS-X algorithm can exhibit a relevant gain due to exploitation of the additional spatial diversity from the direct link, in the same way that CPP-ALS presents a diversity gain over PT2-ALS. Comparing CALS-X with CPP-ALS (and for that matter DALS with PT2-ALS) we can see that there is a coding gain around 5 dB for a BER of 10^{-3}, which is attributed to a second KRST coding at the relay. For the PARATUCK2-based receivers this second coding at the relay is absent. Note that all these receivers are compared under the same transmission rate, as in [11]. Therefore, then the source code length P has been raised for both PT2-ALS and CPP-ALS receivers. In spite of the increased code length at the source, it is possible to reaffirm the conclusion that the nested PARAFAC based receivers (i.e. DALS and CALS-X) present coding gains over the PARATUCK2 based ones (i.e. PT2-ALS and CPP-ALS) for the same transmission rates.

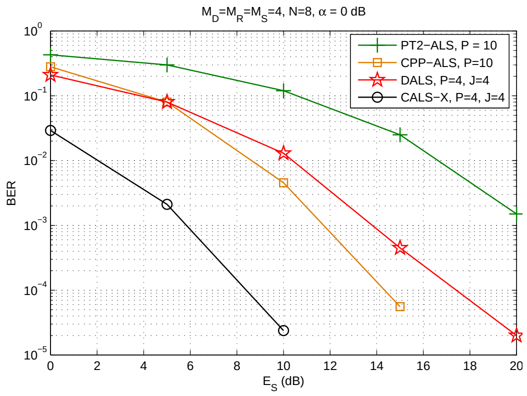

Fig. 2. Impact of the direct link. BER *versus* E_S

B. Impact of the direct link

Undoubtedly, the CALS-X receiver can exploit the increased spatial diversity resulted from combining the two links. This conclusion is readily drawn from the slopes of the BER curves in Fig. 3, which are more pronounced for increased direct link energy (smaller α).

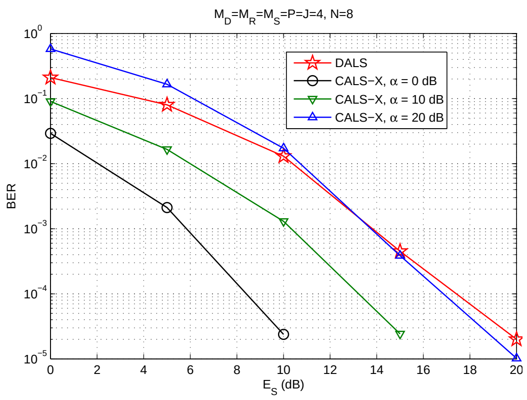

Fig. 3. Impact of α on CALS-X. BER *versus* E_S

On the other hand, the performance of the CALS-X algorithm degrades excessively when the energy of the direct link is very small ($\alpha = 20$ dB). When the direct link is too feeble, it ends up contributing with very distorted signal samples that counteract the benefits of exploiting the stronger relay link. As in [10], combining relay and direct links are interesting when the energy difference of the links is not greater than ~ 10 dB.

V. CONCLUSION

This work has presented a semi-blind tensor-based receiver dedicated to one-way amplify-and-forward cooperative systems that jointly processes signals from relay and direct links to improve symbol and channel estimation. Resorting to a double Khatri-Rao coding at the source and at the relay nodes, this receiver can exploit the signals arriving from the direct link as a PARAFAC model and the signals via relay link as a nested PARAFAC model. Combining these two tensor models, the proposed receiver presented better performance than competing receivers dedicated to the same task.

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