Near ML Uplink Detection for Large Scale MIMO Systems

Alexandre Pereira Junior e Raimundo Sampaio-Neto

Abstract— **Transmission systems known as Massive Multiple-input Multiple-output (MIMO) offer exciting opportunities due to their high spectral efficiencies capabilities. On the other hand, one major issue in these scenarios is the high-complexity detectors of such systems. In this work, we present a low-complexity, near maximum-likelihood (ML) performance achieving detector for the uplink in large multiuser MIMO systems with tens to hundreds of antennas at the base station (BS) and similar number of uplink users subject to antenna correlation and lognormal shadowing channels. The proposed algorithm is derived from the likelihood-ascent search (LAS) algorithm and it is shown to achieve near ML performance as well as to possess excellent complexity attribute. The presented algorithm, termed as random-list based LAS (RLB-LAS), employs several iterative LAS search procedures whose starting-points are in a list generated by random changes in the matched filter detected vector and chooses the best LAS result. Also, a stop criterion is employed in order to maintain the algorithm's complexity at low levels. Near-ML performance detection is demonstrated by means of Monte Carlo simulations and it is shown that this performance is achieved with polynomial complexity on N_t with order less then 2 per symbol, where N_t denotes the total number of uplink users antennas.**

Keywords— **Massive MIMO, LAS detection, Near ML detection.**

I. Introduction

The use of multiple-input multiple-output (MIMO) systems has its roots on the need to achieve greater transmission rates on wireless communications. In order to address the fast growth on the capacity demands experienced in the last years, MIMO techniques were vastly adopted and has become very popular due to the several advantages they offer, including transmit diversity and high data rates [1], [2], [3], being employed at single-user services such as WiFi, WiMAX, and LTE, as well as at multiuser scenarios such as LTE-Advanced and IEEE 802.1 1ac [4], [5]. Besides, the number of antenna elements employed at these systems has been increasing and tends to get even larger due to its potential to achieve extremely-high system throughput [6].

MIMO systems are especially suited when the base station (BS) or access point is equipped with lots of antennas. In such scenarios, massive MIMO systems that achieve extremely high capacity were proposed [6], [7]. In these systems, the number of antenna elements surpasses the order of hundreds antennas. The main issues in realizing such large systems include low-complexity detection and channel estimation. In this work, we approach the complexity of the uplink detection problem in a multiuser MIMO system with large number of antenna elements. Known algorithms such as zero-forcing or minimum mean squared error (MMSE) spatial filtering, sphere decoding [8], and QRM-MLD [9] require matrix inversion which complexity is proportional to the cubed number of antenna elements. In order to reduce this complexity, several MIMO detectors have been proposed based on search strategies such as the Likelihood Ascent Search (LAS) algorithms [10], [11], [12] and search algorithms based on the reactive tabu search [13]. Also, MIMO detectors based on Belief Propagation (BP) strategies [14] and detectors based on simulation strategies such as the Markov Chain Monte Carlo (MCMC) detector [15] were proposed.

Here, we present the Matched Filter Random-List Based LAS (MF-RLB-LAS) algorithm as proposed in [16], that consists on an iterative process composed by several one-stage complex LAS detections. At the first iteration, the LAS procedure starts with the Matched Filter (MF) detection result and, for each of the following iterations, the starting-vector of the LAS procedure is derived from random changes in the MF detection result. The number of iterations is controlled by a stop-criterion that considers the ML cost of the best LAS result so far as well as the first and second-order moments of an error-free decision. The final decision is the LAS search result that achieved the least ML cost. In this work, we extend the results of [16] by considering multiple transmit antenna users, log-normal shadowing and transmit and receive antenna correlation effects. Also, we present a clearer complexity order analysis on the number of required floating-point operations (flops) of several LAS based detectors for this more realistic scenario. Performance results in terms of bit error rate (BER) and computational complexity, evaluated by means of Monte Carlo simulations, are presented and discussed.

The remainder of this paper is organized as follows. After introducing the system model, as well as the considered scenarios, the complex LAS procedure and the MF-RLB-LAS algorithm are shown. Numerical Bit Error Rate (BER) performance and complexity simulation results are presented and discussed at section IV. Finally, at section V, some conclusions are made.

II. System model

Consider a large-scale MIMO system on the uplink consisting of a base station (BS) with N_r antennas and K uplink users with N_{ti} transmit antennas each, where i stands for the user index as depicted in Figure 1. Let $N_t = \sum_{i=1}^{K} N_{ti}$ be the total number of transmit antennas. All users transmit symbols

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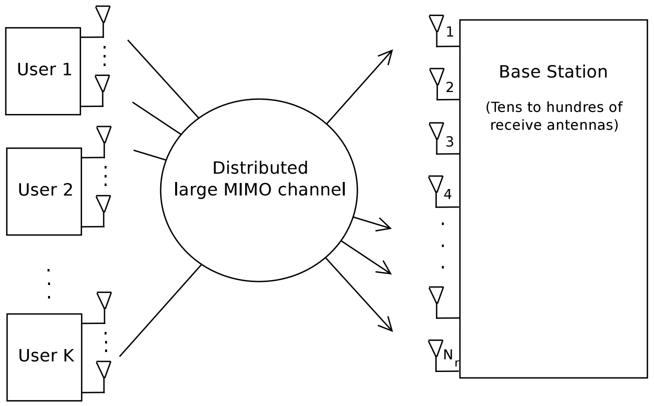

Fig. 1. Representation of MIMO system.

from a modulation alphabet $\mathbb{B}$. It is assumed that the sampled baseband complex signals are available at the BS receiver.

Let $\mathbf{x}_k \in \mathbb{C}^{N_{tk}\times 1}$, $\mathbf{x}_k = [x_{k1}, x_{k2}, \cdots, x_{kN_{tk}}]^T$, be the transmitted symbol vector of the k-th user where x_{ki} is its i-th complex unit energy transmitted symbol. Let $\mathbf{H_k} \in \mathbb{C}^{\mathbf{N_r}\times\mathbf{N_{tk}}}$ be the k-th user channel gain matrix, such that its elements, $h_{n_r,n_{tk}}$, denote the complex channel gain from the n_{tk}-th transmitting antenna to the n_r-th BS receive antenna. In this work, we assume rich scattering conditions so the entries of $\mathbf{H}_k$ can be modeled as i.i.d. $\mathcal{CN}(0,1)$ random variables, where $\mathcal{CN}(0,1)$ stands for complex gaussian distribution with zero mean and unit variance. The received signal vector at the BS, $\mathbf{y}$, can be written as:

$$\mathbf{y} = \mathbf{H}_1\mathbf{x}_1 + \mathbf{H}_2\mathbf{x}_2 + \cdots + \mathbf{H}_K\mathbf{x}_K + \mathbf{n}, \qquad (1)$$

where $\mathbf{n} \in \mathbb{C}^{N_r\times 1}$ is the receiving noise vector, modeled by a complex gaussian random vector with zero mean and covariance matrix $\sigma_n^2\mathbf{I}$. The considered signal-to-noise ratio (SNR) by receiving antenna in dB is $10log_{10}\frac{N_t}{\sigma_n^2}$. The expression on eq. (1) can be more conveniently written as:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \qquad (2)$$

where $\mathbf{H} = [\mathbf{H}_1\mathbf{H}_2\cdots\mathbf{H}_K]$ and $\mathbf{x} = [\mathbf{x}_1^T\mathbf{x}_2^T\cdots\mathbf{x}_K^T]^T$. Also, $(\cdot)^T$ denotes the transpose operator.

Consider $\mathbf{d} \in \mathbb{C}^{N_t\times 1}$ a given detected symbol vector. The ML cost of this vector, $C(\mathbf{d})$, is given by:

$$\begin{aligned} C(\mathbf{d}) &= ||\mathbf{y} - \mathbf{H}\mathbf{d}||^2 \\ &= \mathbf{d}^H\mathbf{H}^H\mathbf{H}\mathbf{d} - 2Real\{\mathbf{y}^H\mathbf{H}\mathbf{d}\}, \end{aligned} \qquad (3)$$

where $||\mathbf{v}||$ denotes the Euclidian norm of $\mathbf{v}$, $(\cdot)^H$ stands for complex Hermitian conjugate, and $Real\{x\}$ stands for the real part of x. The ML solution, $\mathbf{d_{ML}}$, is given by the detected symbol vector that minimizes the ML cost among all possible detected symbol vectors. This minimization problem has exponential complexity in N_t. On the next section, a low-complexity high-performance detection algorithm is presented.

The above described scenario represents uncorrelated and identically distributed communication channels between transmit and receiving antennas subject to Rayleigh fading. This scenario is referred to as scenario A. A more realistic scenario, scenario B, where the effects of correlation and lognormal shadowing were included, was also considered.

In order to add correlation effects, both on transmission and reception, the Kronecker model for communication channels [17] was used, where the channel matrix between user k and the BS, $\mathbf{G}_{0_k} \in \mathbb{C}^{N_r\times N_{tk}}$, is given by:

$$\mathbf{G}_{0_k} = \mathbf{R}_{r_x}^{1/2}\mathbf{H}_k\mathbf{R}_{t_{x_k}}^{1/2}, \qquad (4)$$

where $\mathbf{R}_{r_x}$ and $\mathbf{R}_{t_{x_k}}$ denotes the correlation matrix between the BS antennas at reception and the correlation matrix between the user antennas at transmission, respectively. At equation (4), the operator $(\cdot)^{1/2}$ stands for the following matrix relation:

$$\mathbf{R} = \mathbf{R}^{1/2}\mathbf{R}^{1/2^H}, \qquad (5)$$

The correlation matrices above described are of the form:

$$\mathbf{R} = \begin{bmatrix} 1 & \rho & \rho^4 & \cdots & \rho^{(N_a-1)^2} \\ \rho & 1 & \rho & \cdots & \vdots \\ \rho^4 & \rho & 1 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{(N_a-1)^2} & \cdots & \rho^4 & \rho & 1 \end{bmatrix}, \qquad (6)$$

where N_a is the number of antenna elements and ρ is the correlation index between neighboring antennas.

In order to include the log normal shadowing effects, consider the channel matrix $\mathbf{G}_k \in \mathbb{C}^{N_r\times N_{tk}}$ between the k-th user and the BS [18]:

$$\mathbf{G}_k = \beta_k\mathbf{G}_{0_k}, \qquad (7)$$

where β_k is a lognormal aleatory variable given by:

$$\beta_k = 10^{\frac{\sigma_k\mathcal{N}_k(0,1)}{10}}. \qquad (8)$$

At the above equation, σ_k is the lognormal shadowing spread given in dB and $\mathcal{N}_k(0,1)$ stands for a zero mean unit variance Gaussian aleatory variable.

III. LAS SEARCH AND MF-RLB-LAS ALGORITHM

In this section we present a complex valued alternative of the LAS search procedure proposed in [11] and the MF-RLB-LAS algorithm as in [16].

A. Complex Likelihood Ascent Search Algorithm

This algorithm consists in a series of likelihood-ascent search stages where in each stage a sequence of symbol updates are conduced such that the likelihood cost function monotonically decreases from one iteration to another. At the first stage, the symbol vector updates are performed in just one symbol at a time. Once a local minimum is reached, one could pass to a second stage where two-symbol updates are conduced, then 3-symbol updates and so forth. In order to maintain a low-complexity algorithm, in this work, we consider only one-symbol updates.

At the first stage, starting from an initial solution $\mathbf{d}^{(0)}$, the algorithm searches for a new solution, $\mathbf{d}^{(1)}$, that differs from $\mathbf{d}^{(0)}$ in exactly one position (one-symbol update), such that the ML cost of the new solution is lesser than the ML cost of the current one. The algorithm continues this procedure until a local minimum is achieved, i.e., there is no new solution with lower ML cost that differs from the current solution by one symbol. The ML cost function after the kth iteration is given by $C(\mathbf{d}^{(k)})$ as in eq. (3).

In order to guarantee a monotonic decrease in the ML cost, lets consider the following development:

Assuming that the pth symbol is updated in the $(k+1)$th iteration, $p = 1, 2, \cdots, N_t$, the update rule can be written as

$$\mathbf{d}^{(k+1)} = \mathbf{d}^{(k)} + \lambda_p^{(k)} \mathbf{e}_p, \tag{9}$$

where $\mathbf{e}_p$ denotes the unit vector where its pth entry is one and all the other entries are zero. The possible values of $\lambda_p^{(k)} = d_p^{(k+1)} - d_p^{(k)}$ depend on the constellation being employed and on the current value of the pth entry of the vector $\mathbf{d^{(k)}}$. For example, for 4-QAM constellation such as $[1+j, 1-j, -1-j, -1+j]$, if the pth entry of the vector $\mathbf{d^{(k)}}$ is $d_p^{(k)} = 1+j$, the possible values of $\lambda_p^{(k)}$ are $[-2, -2j, -2-2j]$. Using equations (3) and (9), and defining $\mathbf{G}$ as

$$\mathbf{G} \triangleq \mathbf{H}^H \mathbf{H}, \tag{10}$$

the ML cost difference, $\Delta C_p^{(k+1)} = C_p^{(k+1)} - C_p^(k)$, can be written as

$$\Delta C_p^{(k+1)}(\lambda_p^{(k)}) = |\lambda_p^{(k)}|^2 (\mathbf{G})_{p,p} - 2Real\{\lambda_p^{(k)^*} z_p^{(k)}\}, \tag{11}$$

where $(\mathbf{G})_{p,p}$ denotes the pth entry of the main diagonal of $\mathbf{G}$, $\mathbf{z}^{(k)} = \mathbf{H}^H(\mathbf{y} - \mathbf{H}\mathbf{d}^{(k)})$, and $z_p^{(k)}$ is the pth entry of the $\mathbf{z}^{(k)}$ vector. In each iteration, the value of λ_p is evaluated as

$$\lambda_p^{(k)} = \underset{\lambda \in \mathbb{P}}{argmin} \ \Delta C_p^{(k+1)}(\lambda), \tag{12}$$

where $\mathbb{P}$ is the set of all possible $\lambda_p^{(k)}$ values. To chose the position, s, that will be actually updated at the current iteration, the algorithm considers the position that gives the greater decrease in the cost function, i. e.

$$s^{(k)} = \underset{p \in [1, \cdots, N_t]}{argmin} \ \Delta C_p^{(k+1)}. \tag{13}$$

Other forms of choosing s can lead to different LAS results, as done in the Multiple Search Candidate Sets algorithm (MSCS) [19]. If at a given iteration $\Delta C_s^{(k+1)}$ is not negative, then a local minimum was reached and the algorithm stops.

B. MF-RLB-LAS Algorithm

The key idea behind the RLB-LAS algorithm is to avoid local minima by conducing several iterative complex LAS procedures. In order to maintain the algorithm's complexity at low level, at the first iteration, the MF symbol vector result, $\mathbf{d_{MF}^{(0)}}$, is employed as the starting point vector of the LAS procedure and the LAS result, $\mathbf{d}_{MF}$, is elected as the current decision. For each one of the following iterations, steps 1 to 7 are conduced:

1) Set $\mathbf{d}_m^{(0)} = \mathbf{d}_{MF}^{(0)}$, where m is the iteration index;

2) Chose c randomly from a uniform distribution over $[1, \cdots, N_t]$, where c represents the number of symbols to be changed from the MF result;

3) Select c values sampled uniformly at random, without replacement, from the integers $[1, \cdots, N_t]$, to form the indices, $\mathbf{i} = [\mathbf{i_1}, \cdots, \mathbf{i_c}]$, of the entries of the MF result vector to be changed;

4) For $l = 1, 2, \cdots, c$, select $\mathbf{d}_m^{(0)}(i_l)$ from the symbol constellation $\mathbb{B}$ sampled uniformly at random. Steps 1 trough 4 defines a new starting point symbol vector for the complex LAS procedure of the current iteration;

5) Perform the complex LAS procedure with $\mathbf{d}_m^{(0)}$ as starting-point;

6) If the ML cost of the LAS result symbol vector is less then the ML cost of the current decision, update the current decision as the LAS result of the current iteration;

7) Go to next iteration.

In order to finish the algorithm, a stop-criterion is needed. It must permit a sufficiently large number of iterations in order to find good results in terms of likelihood cost, but, in turn, it must restrict the number of iterations to reduce complexity. The elected criterion is based on a quality metric, named standardized ML cost, evaluated by the difference between the ML cost of the current decision and the average ML cost of an error-free decision, scaled by the standard deviation of the error-free decision ML cost. Note that the ML cost of a error-free detection corresponds to $||\mathbf{n}||^2$, which is Chi-squared distributed with $2N_r$ degrees of freedom with mean $N_r\sigma_n^2$ and variance $N_r\sigma_n^4$. Therefore, the standardized ML cost of the current decision, $\phi(\mathbf{d})$, is given by:

$$\phi(\mathbf{d}) = \frac{||\mathbf{y} - \mathbf{H}\mathbf{d}||^2 - N_r\sigma_n^2}{\sqrt{N_r}\sigma_n^2}. \tag{14}$$

The stop criterion evaluates, at each current decision update, the number of iterations needed to stop the algorithm, N_p, as:

$$Np = \lceil max(c_1\phi(\mathbf{d}), Np_{min}) \rceil, \tag{15}$$

where c_1 and Np_{min} are metric parameters and $\lceil a \rceil$ stands for the least integer greater than a. If, at the end of an iteration, the algorithm has already reached N_p iterations, it chooses the final decision as the current decision and stops.

IV. Numerical Results

The Bit Error Rate (BER) performance and the computation complexity in terms of average number of floating-point operations (flops) of the proposed detectors are analyzed in this section for scenarios A and B. These results were evaluated by Monte Carlo simulations of a 4-QAM modulation. They are an average of 1000 independent simulation runs with 10 symbol vector transmissions per run. Besides the MF-RLB-LAS, the Multiple Input Vector LAS (MIV-LAS) and the Multiple Search Candidate Set LAS (MSCS-LAS) algorithms as presented in [19] were analyzed. The MIV-LAS algorithm performs three LAS search procedures starting from the MMSE detection result, the Zero-Forcing detection result and the MF result, respectively, and chooses as the final decision the result that produced the better ML cost. On the other hand, the MSCS-LAS performs a pre-defined number of LAS procedures, N_l, each one employing a different random update ordering. The average number of flops were computed using the Lightspeed Matlab Toolbox [20].

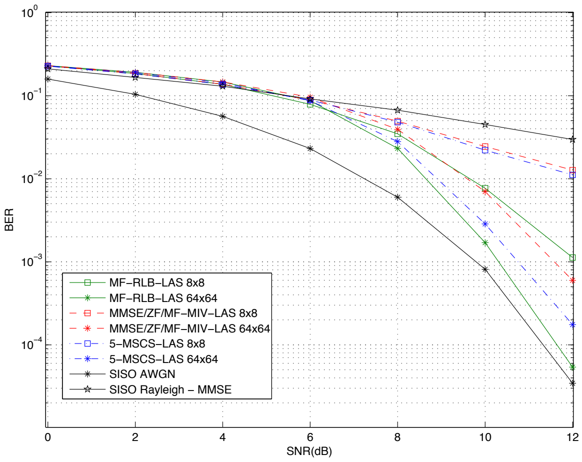

Fig. 2. BER versus SNR curves of MF-RLB-LAS, MMSE/ZF/MF-MIV-LAS and 5-MSCS-LAS detectors for scenario A.

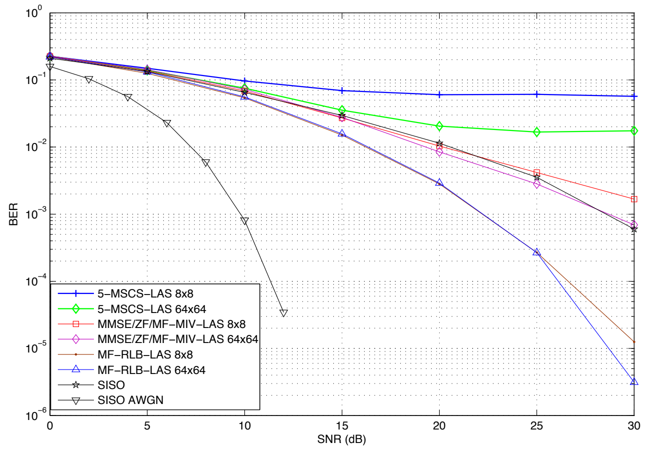

Fig. 3. BER versus SNR curves of MF-RLB-LAS, MMSE/ZF/MF-MIV-LAS and 5-MSCS-LAS detectors for scenario B with $\rho = 0.2$ and $\sigma_k = \sigma = 6dB$.

Figures 2 and 3 present BER per SNR curves for the MF-RLB-LAS, MIV-LAS and MSCS-LAS algorithms on scenarios A and B, respectively. For comparison purposes, BER curves of a Single-Input Single-Output (SISO) system with MMSE detection for the considered scenarios and BER curves of a SISO AWGN channel scenario are also presented. In all cases, the channel matrix is assumed to be known, the number of antennas per uplink user was set to 2 and $N_t = N_r$. The following algorithm parameters were used in the simulations: $N_{p_{min}} = 2, c_1 = 5$ and $N_l = 5$. At scenario B, it was employed $\rho_k = \rho_r = 0.2$ and $\sigma_k = 6dB$. It was evaluated BER curves for $N_t = [8\ 20\ 32\ 64\ 100]$, but, for clarity reasons, only the results for $N_t = N_r = 8$ and $N_t = N_r = 64$ are presented. It can be seen from the results in Fig. 2 that, for scenario A, the BER curves get close to the SISO AWGN result with increasing $N_t = N_r$ for all LAS algorithms considered. This illustrates the capacity of LAS algorithms to approach single antenna AWGN performance even in large MIMO scenario, removing spatial interference from other antennas. Also, it can be noticed that the MF-RLB-LAS achieved the best BER results, meaning that this algorithm approaches SISO AWGN performance faster, i. e., it needs fewer antennas in order to approach SISO AWGN performance. Furthermore, it can be pointed out that, in scenario A, all algorithms achieved better BER results then the SISO with MMSE detection. This indicates the capacity of LAS algorithms to benefit from spatial diversity provided by the MIMO system.

On the other hand, at scenario B, presented in Figure 3, the improve on the BER with increasing number of antenna elements, for the MF-RLB-LAS algorithm, can only be noticed for high SNR values. Despite that, the MF-RLB-LAS still achieves better results then the other algorithms, even if its is compared the MF-RLB-LAS result with $N_t = N_r = 8$ with the result from the others algorithms with increased number of antenna elements. Also, It can be noticed that the MSCS-LAS algorithm was not able to surpass the MMSE SISO performance, even when it was employed $N_t = N_r = 64$.

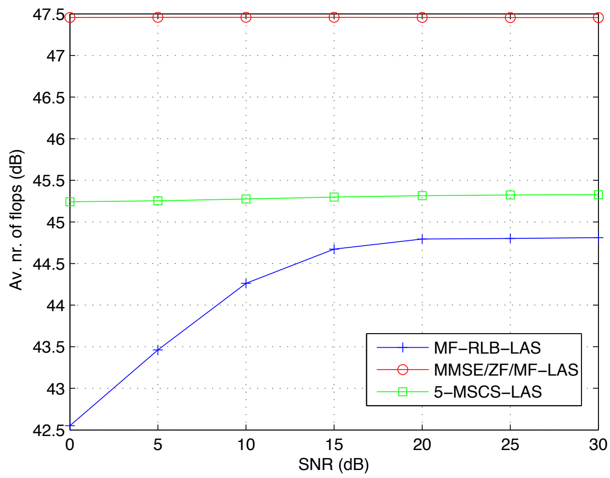

Fig. 4. Average number of floating-point operations by SNR of MF-RLB-LAS, MMSE/ZF/MF-MIV-LAS and 5-MSCS-LAS for a MIMO system with $N_t = N_r = 20$ on scenario B with $\rho = 0.2$ and $\sigma_k = \sigma = 6dB$.

Figure 4 illustrates the complexity results in terms of average number of flops per SNR for algorithms MF-RLB-LAS, MSCS-LAS and MIV-LAS for a MIMO system with $N_t = N_r = 20$ under scenario B conditions. It can be seen that complexity for MF-RLB-LAS, in spite of being lower than the complexity of the other algorithms, increases with the SNR, achieving a maximum value for high SNR. This can be explained by the incapacity of the MF detection result to eliminate spatial interference between transmitted symbols, especially for high SNR values. This leads the MF-RLB-LAS algorithm to perform more iterations at high SNR.

Figure 5 shows complexity curves in terms of the average number of flops per $N_t = N_r$ at scenario B in log-log scale. Markers represent the simulation results while lines are the results of linear curve fits to these simulation results. For the case of MF-RLB-LAS algorithm, the average number of flops for a SNR value that achieved a target BER of 10^{-2} was

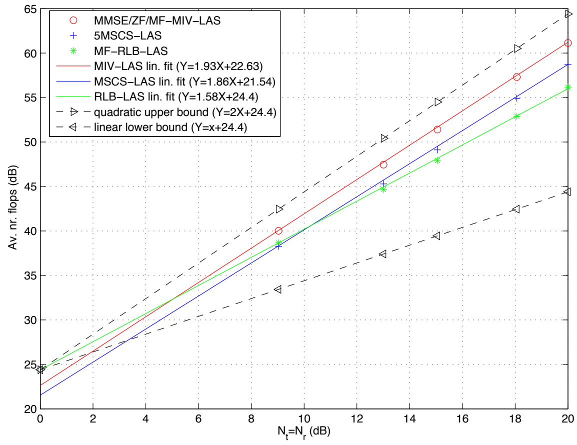

Fig. 5. Average number of floating-point operations by $N_t = N_r$ of MF-RLB-LAS, MMSE/ZF/MF-MIV-LAS and 5-MSCS-LAS for a MIMO system on scenario B with $\rho = 0.2$ and $\sigma_k = \sigma = 6dB$.

considered. Although MSCS-LAS algorithm do not achieve the previous specified target BER for any SNR value, due to the saturation observed in figure 3, its complexity results were presented for comparison. Figure 5 also presents linear and quadratic curves to serve as reference. The results in these curves indicate that the complexity of the analyzed algorithms is polynomial on $N_t = N_r$ with order lesser than 2, since they are straight lines less inclined then the quadratic reference. Also, it can be noticed that the MF-RLB-LAS algorithm has lower computational order then the other two algorithms because its complexity line is less inclined then the lines of the other algorithms.

V. Conclusions

This manuscript extended the analysis of the MF-RLB-LAS detector, a complex LAS based detector for large-scale MIMO systems, for multiple antenna multiuser scenarios where log-normal shadowing and transmit and receive antenna correlation were considered. It was shown that, even for such channels, this detector achieves good BER performance with polynomial computational complexity on the number of transmitting antennas of order less then 2. The MF-RLB-LAS algorithm performs several LAS iterations starting from initial random vectors derived from the matched filter detection result and chooses the LAS result of the iteration that achieves the best ML cost. Monte Carlo simulations were performed in order to obtain BER versus SNR performance curves and computational complexity curves in terms of average number of floating-point operations per SNR and per total number of transmitting antennas for two distinct scenarios: the first one considered only fading effects according to the Rayleigh model. In the second scenario, the effects of transmitting antennas and receiving antennas correlation and lognormal shadowing were incorporated. The performance and complexity results of the MF-RLB-LAS algorithm were compared to the results from the MIV-LAS and MSCS-LAS algorithms, that are also based on the LAS procedure. The proposed MF-RLB-LAS algorithm achieved better performance and complexity results then the other two analyzed algorithms in both scenarios.

References

[1] A.Paulraj, R. Nabar and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge University Press, 2003.

[2] H. Jafarkhani, *Space-Time Coding: Theory and Practice*, Cambridge University Press, 2005.

[3] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*,Cambridge University Press, 2005.

[4] L. Liu, R. Chen, S. Geirhofer, K. Sayana, Z. Shi and Y. Zhou, *Downlink MIMO in LTE-advanced: SU-MIMO vs. MU-MIMO*, IEEE Commun. Magazine, v. 50, nr. 2, pp. 140-147, fev., 2012.

[5] A. Ghosh, R. Ratasuk, B. Mondal, N. Mangalvedhe, T. Thomas,*LTE-advanced: Next-Generation Wireless Broadband Technology*, IEEE Trans. Wireless Commun., v. 17, nr. 3, pp. 10-22, Jun., 2010.

[6] T. L. Marzetta, *Noncooperative Cellular Wireless with Unlimited Numbers of Base Station Antennas*, IEEE Trans. Wireless Commun., v. 9, nr. 11, pp. 3590-3600, nov., 2010.

[7] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors and F. Tufvesson, *Scaling up MIMO: Opportunities and Challenges with Very Large Arrays*, IEEE Signal Process. Mag., v. 30, nr. 1, pp. 40-60, Jan., 2013.

[8] E. Viterbo and J. Boutros, *A Universal Lattice Code Decoder for Fading Channels*, IEEE Trans. Inform. Theory, v. 45, nr. 5, pp. 1639-1642, Jul., 1999.

[9] H. Kawai, K. Higuchi, N. Maeda, M. Sawahashi, T. Ito, Y. Kakura, A. Ushirokawa and H. Seki, *Likelihood Function for QRM-MLD Suitable for Soft-Decision Turbo Decoding and Its Performance for OFCDM MIMO Multiplexing in Multipath Fading Channel*, IEEE PIRMC, pp. 1142-1148, Set., 2004.

[10] Yi Sun, *A Family of Linear Complexity Likelihood Ascent Search Detectors for CDMA Multiuser Detection*, International Symposium on Spread-Spectrum Technology and Applications, pp. 713-717, 2000.

[11] K. Vishnu Vardhan, Saif K. Mohammed, A. Chockalingam and B. Sundar Rajan, *A Low-Complexity Detector for Large MIMO Systems and Multicarrier CDMA Systems*, IEEE Journal on Selected Areas in Communications, v. 26, nr. 3, pp. 473-485, Abr., 2008.

[12] Saif K. Mohammed, A. Chockalingam and B. Sundar Rajan, *A Low-Complexity Near-ML Performance Achieving Algorithm for Large MIMO Detection*, IEEE International Symposium on Information Theory, pp. 2012-2016, 2008.

[13] N. Srinidhi, Saif K. Mohammed, A. Chockalingam and B. Sundar Rajan, *Low-Complexity Near-ML Decoding of Large Non-Orthogonal STBCs using Reactive Tabu Search*, International Symposium on Information Technology - ISIT 2009, pp. 1993-1997, Jul., 2009.

[14] Wataru Fukuda, Takashi Abiko, Toshihiko Nishimura, Takeo Ohgane, Yasutaka Ogawa, Yusuke Ohwatari and Yoshihisa Kishiyama, *Low-complexity Detection Based on Belief Propagation in a Massive MIMO System*, IEEE VTC, pp. 1-5, 2013.

[15] T. Datta, N. A. Kumar, A. Chockalingam and B. S. Rajan, *A Novel MCMC Algorithm for Near-Optimal Detection in Large-Scale Uplink Mulituser MIMO Systems*, IEEE Information Theory and Applications Workshop (ITA), pp. 69-77, Fev., 2012.

[16] Alexandre A. Pereira Jr and Raimundo Sampaio_Neto, *A Random-List Based LAS Algorithm for Near-Optimal Detection in Large-Scale Uplink Multiuser MIMO Systems*, Proceedings of 19th International ITG Workshop on Smart Antennas - WSA 2015, pp. 1-5, Mar., 2015.

[17] J. P. Kermoal, L. Schumacher, and K. I. Perdersen et al, *A Stochastic MIMO Radio Channel Model with Experimental Validation*, IEEE Journal on Selected Areas on Communications, v. 20, nr. 6, pp. 1211-1226, Ago., 2002

[18] Theodore S. Rappaport, *Wireless Communications - Principles and Practice*, Prentice Hall PTR, 1996.

[19] Peng Li and Ross D. Murch, *Multiple Output Selection-LAS Algorithm in Large MIMO Systems*, IEEE Communications Letters, v. 14, nr. 5, pp. 2012-2016, Maio, 2010.

[20] T. Minka, *The UpshapeLightspeed UpshapeMatlab Toolbox, Efficient Operations for UpshapeMatlab Programming, version 2.2*, Microsoft Corp., 2007.