

# Improvement on Sampling Clock Offset Estimation for Mobile OFDM Systems

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**Abstract**—Sampling Clock Offset (SCO) estimation is an important issue in Orthogonal Frequency-Division Multiplexing (OFDM) systems because sampling frequency mismatch between the transmitter and the receiver may severely degrade the system performance due to the loss of orthogonality between the subcarriers. SCO estimation in mobile environment is quite challenging since channel variation leads to an additional phase rotation that masks the SCO effects. However, most of the existing techniques rely on the assumption of a time-invariant channel and become considerably inaccurate in a mobile environment. In this paper, we propose an improvement to an existing pilot-aided SCO estimator aiming to provide robustness to channel variations. Performance was evaluated through simulations in a ISDB-T (Integrated Services Digital Broadcasting Terrestrial) compliant system, and the results have shown a considerable RMSE reduction for all ranges of SNR, specially for higher Doppler spread.

**Keywords**—OFDM, Sampling Clock Offset, Sampling Frequency Offset, Time-varying Channels, Doppler spread, ISDB-T system.

## I. INTRODUCTION

Orthogonal Frequency-Division Multiplexing (OFDM) is a multi-carrier technique that achieves high spectrum efficiency by dividing data into closely spaced subcarriers which constitute parallel orthogonal data-streams. It has been adopted in several broadband communication systems such as digital TV standards DVB [1] and ISDB [2].

It is known that the performance of OFDM systems is very sensitive to a precise synchronization. Sampling Clock Offset (SCO), also called Sampling Frequency Offset (SFO), occurs when the receiver's sampling frequency is not perfectly adjusted. It is a central issue in practical implementations since such error leads to the loss of orthogonality between the subcarriers, or Inter-Carrier Interference (ICI), causing performance degradation. The effects are intensified when larger OFDM symbols are used, as it is the case on digital TV standards.

A variety of SCO estimators have been proposed so far [3]–[9], the Post-FFT methods being more widespread than the pre-FFT ones. Most of the post-FFT estimators use the pilots of adjacent symbols to identify the phase rotation caused by the SCO [3]–[6]. In those methods, the channel is assumed to be time-invariant or at least slow time-variant, so that its influence can be cancelled between different symbols. A different pilot-based post-FFT technique proposed in [7] tries to detect the shift on the estimated channel impulse response

(CIR) caused by SCO. Although it is designed to work in time-variant channels, it might result in a low resolution estimation depending on system parameters. Pre-FFT methods have been proposed in [8], [9]. They perform a blind estimation by relying on the statistical particularities of the time-domain OFDM signal, typically using the cyclic prefix. But, the method in [8] is not suited to multipath channels, and both of them, similar to the previous ones, cannot cope with time-varying channels.

Based on a conventional pilot-aided SCO estimator, we propose an improved algorithm that mitigates the effects of time-varying channels. Note that the underlying algorithm is still affected by the channel variations, but we establish an heuristic to identify the most distorted data and prevent them from degrading the estimation.

This paper is organized as follows. In section II the OFDM system model is described, as well as the effects of the SCO on the received signal. Section III is divided in two parts: first, we briefly present the conventional pilot-aided SCO estimator in which our proposal is based; then, we describe the proposed estimator that aims for stable operation on mobile environment. The simulation results and analysis are presented in section IV and conclusions in section V.

## II. SYSTEM MODEL

Consider an OFDM system which transmitted baseband signal is given by [10]:

$$s(t) = \frac{1}{\sqrt{T_u}} \sum_{l=-\infty}^{+\infty} \sum_{k=-K/2}^{+K/2} a_{l,k} \psi_{l,k}(t) \quad (1)$$

where  $a_{l,k}$  denotes the data symbols with rate  $1/T_u$  to be sent, with  $k$  being the subcarrier frequency index and  $l$  being the OFDM symbol time index,  $K$  is the total of data symbols on each OFDM symbol and  $\psi_{l,k}(t)$  denotes the subcarrier pulses with baseband frequency  $f_k = k/T_u$ .

We also suppose that the OFDM symbol has a guard interval of length  $T_g$  appended to its suffix to retain the orthogonality after passing through the time dispersive channel, so that we can write the subcarriers as:

$$\begin{aligned} \psi_{l,k}(t) &= \exp\left(j2\pi\left(\frac{k}{T_u}\right)(t - T_g - lT_s)\right) u(t - lT_s) \\ u(t) &= \begin{cases} 1, & 0 \leq t < T_s \\ 0, & \text{else} \end{cases} \end{aligned} \quad (2)$$

where  $T_s = T_u + T_g$  is the resulting symbol length, which is equivalent to  $N_s = N + N_g$  samples for a sampling rate of  $1/T_u$ .

The signal is transmitted over a frequency selective fading channel and received by an anti-aliasing filter used to sample the signal. In such a case the equivalent channel impulse response is time-varying and can be represented by:

$$h(\tau, t) = \sum_i h_i(t) \delta(\tau - \tau_i) \quad (3)$$

where  $\tau_i$  is the delay of the  $i$ th path and  $h_i(t)$  is a complex Gaussian stochastic process with a variance  $\sigma_{h_i}^2$  and correlation described by the Jakes' model [11].

The received signal is

$$r(t) = \sum_i h_i(t) s(t - \tau_i) \quad (4)$$

which is then passed through an appropriate anti-alias filter and sampled at timing instants  $t_n = nT'$ , where  $T'$  is the sampling period at the receiver, yielding

$$r(t_n) = \sum_i h_i(nT') s(nT' - \tau_i) + \eta(nT') \quad (5)$$

where  $\eta(nT')$  is assumed to be an additive complex white Gaussian noise with variance  $\sigma_\eta^2$ .

After removing the guard interval, the  $l^{th}$  received symbol is represented by  $N$  samples

$$r_{l,k} = r((n + N_g + lN_s)T') = r(n'T') \quad (6)$$

with the abbreviation  $n' = n + N_g + lN_s$ .

In case of perfect synchronization ( $T' = T$ ) and assuming the channel to be constant during the transmission one OFDM symbol, the demodulation of the signal through DFT gives

$$z_{l,k} = a_{l,k} H_{l,k} + \eta_{l,k} \quad (7)$$

where  $H_{l,k}$  is the channel frequency response (CFR) given by the DFT of the correspondent CIR  $\{h_i(l) \exp(-j2\pi\tau_i l/N)\}$ .

*a) Fast-Fading Channel:* If the channel varies within one OFDM symbol, the frequency domain received signal can be represented by

$$z_{l,k} = a_{l,k} H_{l,k} + \eta_{l,k} + \eta_{ICI;l,k} \quad (8)$$

The time variance breaks the orthogonality causing ICI, which from now on will be modeled as additive noise  $\eta_{ICI}$ . Strictly speaking,  $H_{l,k}$  no longer represents the physical channel at a certain time instant but rather the result of an averaging over a time period.

*b) Sampling clock offset:* In the case where  $T' \neq T$ , we can define the relative sampling clock offset (SCO) as  $\zeta = (T' - T)/T$ . We suppose that no other synchronization error is present. By substituting (1) in (5), the time domain signal may be written as

$$r_{l,n} = \sum_i h_i(nT') \sum_l \sum_k a_{l,k} \psi_{l,k}(n'T' - \tau_i) + \eta_{l,n} \quad (9)$$

After the demodulation via FFT, the frequency domain samples become [10]

$$\begin{aligned} z_{l,k} = & [ \exp(j\pi\phi_k) \cdot \exp(j2\pi\phi_k(lN_s + N_g)/N) ] \\ & \cdot \text{sinc}(\pi\phi_k) a_{l,k} H_{l,k} \\ & + \sum_{l \neq k} [ \exp(j\pi\phi_{i,k}) \cdot \exp(j2\pi\phi_i(lN_s + N_g)/N) ] \\ & \cdot \text{sinc}(\pi\phi_{i,k}) a_{l,i} H_{l,i} + \eta_{ICI;l,k} + \eta_{l,k} \end{aligned} \quad (10)$$

where

$$\begin{aligned} \phi_{i,k} &= (1 + \zeta)i - k \\ \phi_k &= \phi_{k,k} = \zeta k \end{aligned} \quad (11)$$

A first remarkable effect is the appearance of new additive ICI term, *i.e.*, the second term of (10), which for simplicity we will be incorporated into the noise term  $\eta_{ICI}$  from the time-varying channel scenario.

Some further simplifications can be performed on the previous equation. For practical deviations ( $\phi_k \ll 1$ ) the multiplicative term  $\text{sinc}(\pi\phi_k) \approx 1$  and can be omitted. Likewise, the time invariant term  $\exp(j\pi\phi_k)$  can be incorporated to the channel gain factor  $H_{l,k}$ . We are left with

$$z_{l,k} = a_{l,k} H_{l,k} \exp\left(\frac{j2\pi k \zeta}{N}(lN_s + N_g)\right) + n_{l,k} + n_{ICI;l,k} \quad (12)$$

Therefore, the main effect of SCO on the received symbols – set aside ICI – is a phase rotation that linearly grows on the subcarrier index  $k$ . The slope of this linear phase distortion is directly proportional to the relative SCO ( $\zeta$ ). For this reason, this is the parameter that the SCO tracking algorithm presented on section III will try to estimate.

### III. SCO ESTIMATION

Pilot symbols are used on the estimation process. In this paper we adopt a pilot structure compatible with ISDB-T systems, which is represented in figure 1. Each OFDM symbol may contain one of the four different types of scattered pilots – A, B, C and D in sequence – characterized by the subcarrier index of the first pilot. Within one symbol the pilots are equally spaced by 12 subcarriers. The pilot content is a BPSK (Binary Phase-Shift Keying) symbol following a PRBS (Pseudo Random Binary Sequence) defined in [2].<sup>1</sup>

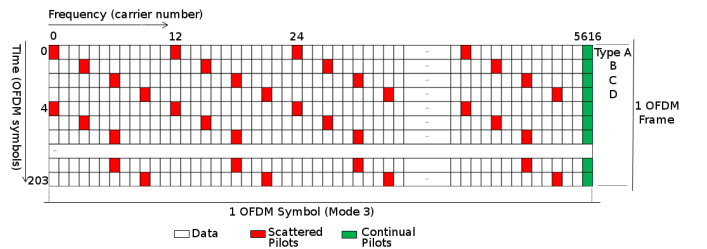


Fig. 1. Pilot pattern on ISDB-T systems.

<sup>1</sup>Nevertheless, the following algorithms are not restricted to this specific pilot pattern, being applicable to other pilot spacings both in time and frequency.

### A. Conventional Method

Considering the equation (12) and neglecting the noise terms, it remains impossible to estimate the phase rotation on the pilot symbols because of the unknown channel gain term  $H_{l,k}$ . As a matter of fact, in most practical systems the synchronization block precedes the channel estimation.

The main idea of the method proposed in [4] is to eliminate the effects of the channel gain term by taking the phase difference between two pilots in different symbols that hold the same subcarrier index. This can be achieved by taking the product with the complex conjugate of the second pilot.

$$\begin{aligned} Y_{D,k} &= z_{l+D,k} \cdot z_{l,k}^* \\ &= [a_{l+D,k} H_{l+D,k} \exp(j2\pi k \zeta ((l+D)N_s + N_g)/N)] \\ &\quad \cdot [a_{l,k}^* H_{l,k}^* \exp(-j2\pi k \zeta (lN_s + N_g)/N)] \\ &= |a_{l,k}|^2 |H_{l,k}|^2 \exp(j2\pi k \zeta DN_s/N) \end{aligned} \quad (13)$$

where  $D$  is the distance in symbols between the pilots.

In ISDB-T pilot structure we choose  $D=4$ , so that  $a_{l,k}=a_{l+D,k}$ . Also note that in (13) the channel is assumed unchanged between the two symbols where the pilots are located, i.e.  $H_{l,k}=H_{l+4,k}$  so that the product  $H_{l+4,k} H_{l,k}^* = H_{l,k} H_{l,k}^* = |H_{l,k}|^2$ . Furthermore, the phase of  $Y_{D,k}$  no longer depends on the symbol index, but only on the distance  $D$ .

If we plot the phase of the products  $Y_{D,k}$  as a function of the pilot subcarrier index  $k$ , as long as the static channel hypothesis holds, a linear pattern with some noise is expected as in figure 2.

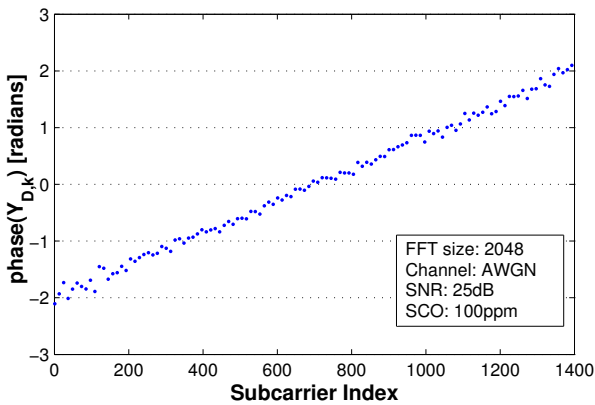


Fig. 2. Phase of the product  $Y_{D,k}$  with a 25dB SNR

The main goal is to determine the slope, which directly gives the corresponding  $\zeta$  (SCO). In order to provide such estimate, let us consider the phase difference between two distinct arbitrary carriers  $k_1$  and  $k_2$ , with  $\Delta k = k_1 - k_2$ , so that

$$\Delta B(D, \Delta k) = \arg(Y_{D,k_1} \cdot Y_{D,k_2}^*) = 2\pi \Delta k \zeta DN_s/N. \quad (14)$$

Now, considering the presence of noise that was neglected in eqs. (13) and (14), the best way to achieve a more robust estimation is to consider all possible combinations of two  $Y_{D,k}$  and then calculate the average - always dividing by the corresponding carrier spacing  $\Delta k$

$$\hat{\zeta} = \frac{N}{2\pi DN_s} \left( \frac{\sum_{\Delta k} \Delta B(D, \Delta k) / \Delta k}{M(M-1)/2} \right) \quad (15)$$

where  $M$  is the total number of pilots in one OFDM symbol.

This is a *one-shot* estimator, and a new estimation can be obtained at every incoming symbol.

However, as mentioned before, one of the assumptions used in this method is the channel invariance on the interval of  $D$  OFDM symbols. If such assumption is not met, the channel variation will cause an extra phase rotation that can be different for each subcarrier and, in principle, cannot be distinguished from the SCO effect. As shown in figure 3 the referred linear pattern on the phase of  $Y_{D,k}$  might get severely distorted.

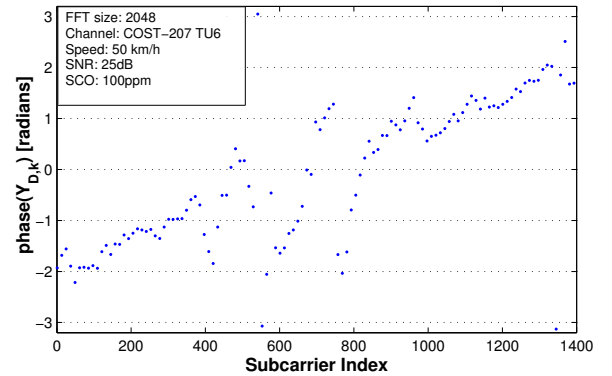


Fig. 3. Phase of the product  $Y_{D,k}$  on TU6 channel 50 km/h and 25dB SNR

To illustrate this point, figure 4 shows the output of the aforementioned method for a constant SCO value of  $\zeta = 100$  ppm and under a mobile channel TU6 (see table II) with a normalized Doppler frequency of  $f_d T_s = 0.038$  which, for a carrier frequency  $f_c = 806$  MHz, corresponds to 200 km/h, 100 km/h and 50 km/h respectively for FFT size 2048, 4096 and 8192 on ISDB-T standard. It can be seen that the channel variation induces severe deviations on the output of the estimator.

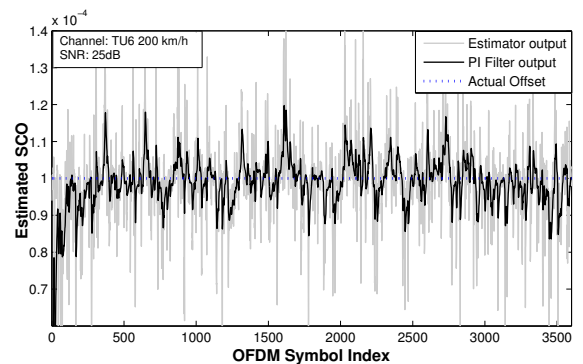


Fig. 4. Conventional output of the estimator (eq. (15)) on a TU6 channel 200 km/h (ISDB-T,  $N=2048$ )

An effective strategy is to post-process the estimator output by using a Proportional-Integral (PI) filter [3]. It guarantees a smoother result, since the present estimation is averaged with the filter state which contains information from all the past estimations. The PI filter output is also plotted in figure 4 and we can see that, although better behaved, its output is still unstable on a mobile environment.

### B. Mitigation of Time-varying Channel Effects

The phase distortion caused by the channel variation is different on each subcarrier. Some might be more distorted than others. For instance, in figure 3 the central subcarriers have been much more affected than the rest. This observation is the central idea of the improvement we are proposing. Simply put, the method consists of a heuristic to identify the probable outliers – the most distorted terms – thus preventing them from contaminating the estimation.

In equation (15) all possible combinations of  $Y_{D,k}$  are considered on the averaging, even the ones that involve an outlier. Instead, we can set a certain threshold  $\delta$  around a reference value and ignore all the  $\Delta B(D, \Delta k) / \Delta k$  that would lead to a result outside this zone.

Let

$$\hat{\zeta}(\Delta k) = \frac{N}{2\pi D N_s} \frac{\Delta B(D, \Delta k)}{\Delta k} \quad (16)$$

be the estimation given by a particular pair  $(k_1, k_2)$ .

Then, the summation in (15) is replaced by a summation over the set  $\Omega = \{\Delta k : |\hat{\zeta}(\Delta k) - \zeta_{ref}| < \delta\}$ , giving

$$\zeta_{est} = \frac{N}{2\pi D N_s} \cdot \left( \frac{\sum_{\Omega} \Delta B(D, \Delta k) / \Delta k}{|\Omega|} \right) \quad (17)$$

where  $|\Omega|$  is the cardinality of the set  $\Omega$ .

The reference value  $\zeta_{ref}$  should be an approximation of the actual offset<sup>2</sup>. We propose to use the PI filter output for this purpose. It should be a robust choice since it ponders all the past estimations. Figure 5 illustrates the threshold application.

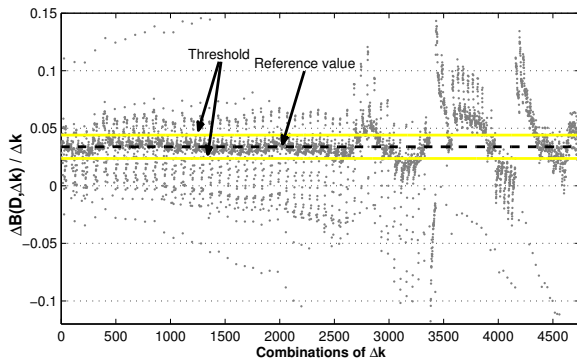


Fig. 5. Threshold application on the values of  $\Delta B(D, \Delta k)$

An additional protection mechanism might be considered in the case where the offset changes abruptly and falls beyond the threshold bounds. In order to prevent the disposal of (almost) all data, one could for example: momentarily increase the threshold or saturate the outliers instead of discarding them.

Since the PI filter presents an initial transient, it is necessary to make sure that the PI filter output has already reached the steady-state before start applying the threshold. The total required time depends on the filter's *forgetting factor* and can be easily adjusted experimentally.

For comparison, figure 6 shows the outputs of the proposed estimator under the exact same conditions as in figure 4. The accuracy improvement is evident.

<sup>2</sup>Naturally, the closest it is from the actual offset, the better the results.

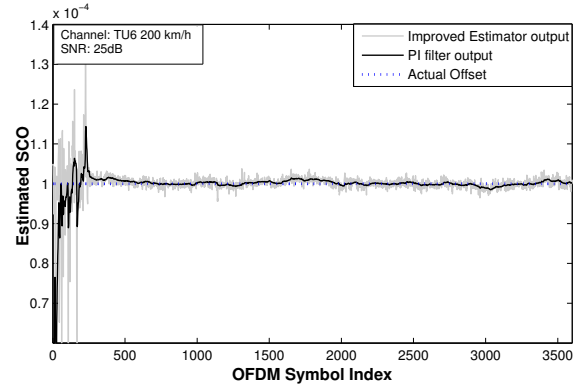


Fig. 6. Proposed estimator's output on TU6 channel 200 km/h (ISDB-T,  $N = 2048$ ), with PI forgetting factor 0.9 and threshold  $\delta = 6$  ppm.

A similar idea has been proposed in [12], with a central difference on the choice of the reference value. They performed an initial estimation by the conventional method (given by equation (15)) and took it as a reference for identifying the outliers on that same symbol. Then, the outliers are eliminated and the estimation is recalculated.

However, on symbols where most subcarriers are severely distorted the classical estimation will be very far from the actual SCO. So, taking it as a reference would imply on a poor identification of the outliers. This problem is avoided when using the PI filter output as the reference since it contains not only the information of the present symbol – assumed to be strongly corrupted – but also accumulates the information of all previous symbols.

Section IV presents some simulation results confirming the superiority of our method. Though, it is important to note that [12] proposes other measures to help improve the estimation performance which we have not included in our simulations (for instance, the noise reduction method). We have only implemented the part that corresponds directly to our proposition, so that the comparison is direct.

## IV. SIMULATION RESULTS

The simulations parameters based on the ISDB-T standard are listed in Table I. The mobile channel profile is the TU6 (6-tap Typical Urban) defined by COST-207 standard [13] and described in Table II.

TABLE I  
SIMULATION PARAMETERS

FFT size	2048
Guard interval	1/8
System bandwidth	6 MHz
Carrier Frequency	806 MHz
Data Modulation	64QAM
Convolutional Code Rate	2/3
PI filter forgetting factor	0.90
Channel Profile	TU6 (see Table II)
Threshold ( $\delta$ )	6 ppm

The performance of the algorithms were evaluated by means of the RMSE (root mean square error) of the estimations over time. We have compared three techniques: the conventional

TABLE II  
TU6 CHANNEL PARAMETERS

#Tap	Delay [us]	Power [dB]
1	0.0	-3
2	0.2	0
3	0.5	-2
4	1.6	-6
5	2.3	-8
6	5.0	-10

method (described in section III-A), the proposed improvement (described in section III-B) and the technique proposed by Won et al. [12].

We have performed simulations in different mobile speeds by using ten independent channel realizations with a total of 3600 OFDM symbols each. Figures 7 and 8 show the RMSE performance as a function of the SNR for the mobile speed of 50 km/h (Doppler spread  $f_d = 37.31$  Hz and normalized Doppler spread  $f_d T_s = 0.0094$  for the specified parameters) and 200 km/h ( $f_d = 149.26$  Hz and  $f_d T_s = 0.0376$ ).

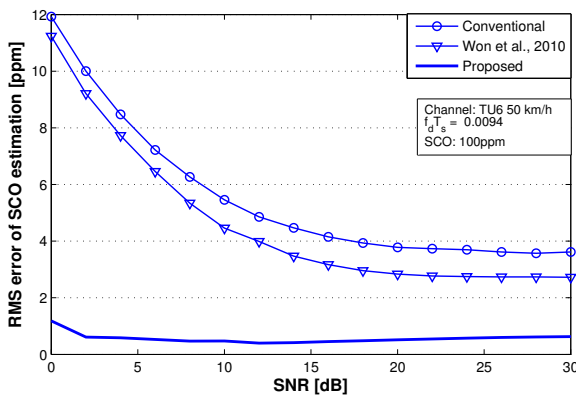


Fig. 7. RMS error vs. SNR (TU6 channel 50 km/h)

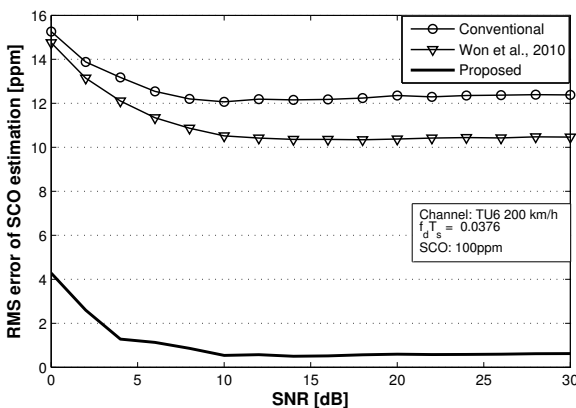


Fig. 8. RMS error vs. SNR (TU6 channel 200 km/h)

We can see that the conventional method performance is severely degraded as Doppler spread increases. The proposed method proved to effectively mitigate the channel variation effects achieving a significantly better performance even when compared to the method proposed in [12]. For higher Doppler spreads the gap between the methods increases.

## V. CONCLUSION

In this paper we proposed an improvement to a conventional pilot-based SCO estimation algorithm aiming the mobile environment. In this scenario some pilots may suffer severe phase distortion, degrading the estimation. Our proposal consists of a heuristic to identify the least reliable data and eliminate them from the calculation of the SCO. Simulation results have shown a considerable RMSE reduction for all ranges of SNR, specially for higher Doppler spread.

A perspective for future works would be a more detailed theoretical development on the proposed procedure, in order to better understand its capabilities and limitations.

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