Channel Estimation for Two-Way Relay Networks Over Frequency Selective Fading Channels

Pedro Ivo da Cruz and Murilo Bellezoni Loiola

***Abstract*— Two-way relay networks can have their throughputs improved by adopting the use of physical-layer network coding. To work properly, these systems need to know the channel impulse response. Most previous works on channel estimation for physical-layer network coding systems consider flat fading or frequency-selective fading with orthogonal frequency division multiplexing modulation. However, for single carrier systems under frequency-selective channels these estimators can not be applied directly. Therefore, a least squares channel estimator for these conditions is proposed here. Simulations are performed to evaluate the performance of this channel estimator and the results show the effectiveness of the proposed technique.**

***Keywords*— physical layer network coding, analog network coding, amplify-and-forward, two-way relay network, channel estimation, least squares**

I. Introduction

The first discussion about communication systems that use two-way channels for exchanging information between two nodes was presented by Shannon in 1961 [1]. This subject has gained a renewed interest when a communication system that uses a technique called Physical-layer Network Coding (PNC) was proposed in 2006 [2], [3]. It adopts a Two-way Relay Network (TWRN) and allows two nodes to send data simultaneously to the relay, improving the throughput of the network [3].

The PNC systems take advantage from the electromagnetic interference between the signals sent by two or more nodes at the same time and frequency. Specifically in the TWRN, two nodes send information simultaneously to the relay node, that must perform the PNC mapping [2] in order to transform its received signals originated from both users into one that can be recognized by the end nodes. It is shown in [4] that we can consider the signal interference itself as a PNC mapping, and, therefore, the relay only needs to amplify the received signal and send it to the end nodes. This protocol is known as Amplify-and-Forward (AF), and the PNC systems that use this protocol are called Analog Network Coding (ANC), once the relay does not need to perform any detection, and the PNC mapping occurs in the analog domain.

Most works on PNC systems considers that the channel impulse response (CIR) of each channel are perfectly known by the relay and end nodes. However, in practical situations, these CIR are not known a priori. Hence, channel estimation techniques are essential for practical deployment of PNC systems. Because of the easy implementation of the ANC systems, most works of channel estimation focus on these systems. Furthermore, it can be shown that for ANC systems, the channel estimation is only necessary at the end nodes for detection, and it is not necessary the knowledge of the individual channels, just the impulse responses of the cascaded channels [5]–[9].

The majority of the works about channel estimation in PNC systems considers only flat fading environments [5], [6], and when dealing with frequency-selective fading environments, adopts orthogonal frequency division multiplexing (OFDM) modulation [7]–[9], since it is a robust technique against frequency selectivity of wireless channels. Specifically, an optimal channel estimation and training design for flat-fading channels was proposed in [6], while for frequency-selective fading environments, a least squares (LS) channel estimation technique for OFDM modulated ANC was proposed in [7].

However, there are single carrier (SC) communications systems where PNC technique could be applied, such as GSM [10], and satellite networks with CDMA [11]. Therefore, this work proposes an LS channel estimation technique for ANC systems using a SC modulation over frequency-selective fading channels.

This paper is organized as follows: section II presents the system model that is used in this work; section III develops the proposed least squares channel estimator; while simulation results are presented in section IV; finally, conclusions are made in section V.

II. System Model

A simple TWRN typically has two source nodes (1 and 2) and one relay node (R), as shown in Figure 1, where node 1 sends information to node 2, and vice versa, using the node R to assist their communications.

The traditional network would need four transmission stages to exchange data between nodes 1 and 2: the first stage is used to send data from node 1 to node R; the second, to send this data from node R to node 2; the third, to send data from node 2 to node R; and the fourth one to send data from node R to node 1.

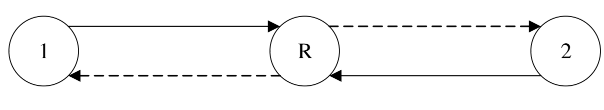

Fig. 1. Transmission stages in a TWRN.

When applying the PNC technique, it only takes two transmission stages to exchange data between the two source nodes. In this case, nodes 1 and 2 send their information

Pedro Ivo da Cruz and Murilo Bellezoni Loiola are with the Engineering, Modeling and Applied Social Sciences Center, Federal University of ABC, Santo André, SP, Brazil, E-mails: pedro.cruz@ufabc.edu.br, murilo.loiola@ufabc.edu.br. This work was partially supported by CAPES and by FAPESP under grant 2013/25977-7.

simultaneously to the node R. This stage is called Multiple Access (MAC). Let $x_i(n)$ be the baseband symbol sent by the node i at the discrete time instant n, $h_{iR}(n)$ be the CIR between the nodes i and R, $w_R(n)$ be the additive white Gaussian noise (AWGN) with distribution $\mathcal{N}(0, \sigma_W^2)$ at node R, and $*$ denotes the convolution operation. The discrete-time and baseband signal received at R can be written as:

$$y_R(n) = x_1(n) * h_{1R}(n) + x_2(n) * h_{2R}(n) + w_R(n). \quad (1)$$

In the next stage, R amplifies the signal by a factor α and forwards the resulting signal to both nodes 1 and 2. This factor can be used to adjust the power of transmitted signal at the relay. This stage is called Broadcast (BC). Due to the symmetry of this network, the performance can be analyzed just at node 1, since it is equivalent for the node 2. Let $h_{Ri}(n)$ be the CIR between nodes R and i, $w_1(n)$ is AWGN at the node 1, also with distribution $\mathcal{N}(0, \sigma_W^2)$. The signal received at the node 1 at the BC stage can be written as:

$$y_1(n) = \alpha y_R(n) * h_{R1}(n) + w_1(n). \quad (2)$$

The cascaded channels can be defined as $a(n) = h_{1R}(n) * h_{R1}(n)$, $b(n) = h_{2R}(n) * h_{R1}(n)$, and $w(n) = \alpha w_R(n) * h_{R1}(n) + w_1(n)$. Then, substituting (1) in (2), the signal received at the node 1 at the BC stage can be rewritten as:

$$y_1(n) = \alpha x_1(n) * a(n) + \alpha x_2(n) * b(n) + w(n). \quad (3)$$

Detection can be performed as shown in (4). It means that, given $y_1(n)$, $x_1(n)$, $a(n)$ and $b(n)$, it is possible to obtain an estimate of $x_2(n)$ through maximum likelihood data detection, i.e., the estimate will be the value of x_2 that minimizes the squared of the absolute difference between the received signal $y_1(n)$ and a version of the transmitted signal $\alpha y_R(n)$ without noise interference.

$$\hat{x}_2(n) = \arg\min_{x_2(n)} |y_1(n) - \alpha x_1(n) * a(n) - \alpha x_2(n) * b(n)|^2 \quad (4)$$

A practical way of performing this at node 1 is to extract its self-information from $y_1(n)$, as shown in (5). However, to perform this task correctly, the knowledge of $a(n)$ is necessary.

$$\begin{aligned} \tilde{x}_2(n) &= y_1(n) - \alpha x_1(n) * a(n) \\ &= \alpha x_2(n) * b(n) + w(n). \end{aligned} \quad (5)$$

Then, the signal $\tilde{x}_2$ needs to be equalized. Linear equalizers such as the Zero-forcing (ZF) equalizer or algorithms such as the Maximum Likelihood Sequence Estimation (MLSE) can be used for this purpose [10].

The model described above shows the importance of estimating the cascaded CIR $a(n)$ and $b(n)$ at the end nodes for the self-information extraction and equalization. It also shows one advantage of the ANC scheme: it only needs the knowledge of the CIR at the end nodes, and not at the relay node.

III. Least Square Channel Estimation

As mentioned in Section I, most works on channel estimation for PNC systems consider frequency-selective channels with OFDM modulation. In this work an LS estimator in time domain is proposed to estimate frequency-selective and time invariant CIR for PNC systems using SC transmission.

Assuming that the nodes 1 and 2 send N training symbols $m_1(n)$ and $m_2(n)$, respectively, (3) can be interpreted as a linear model that can be written in matrix form. To do so, a matrix $\mathbf{M}$ can be defined as:

$$\mathbf{M} = [\mathbf{M}_1 \ \mathbf{M}_2], \quad (6)$$

where $\mathbf{M}_1$ and $\mathbf{M}_2$ are the convolution matrices containing the training symbols sent by nodes 1 and 2, respectively. $\mathbf{M}$ has dimension $N + 2(N_{CH} - 1) \times 2(2N_{CH} - 1)$, where N_{CH} is the channel length, considered the same for all channels, although the estimator can be easily generalized for different channel lengths.

Let $\mathbf{h}$ be the vector that contains the coefficients of both channels as:

$$\mathbf{h} = \begin{bmatrix} \mathbf{a} \\ \mathbf{b} \end{bmatrix}, \quad (7)$$

where $\mathbf{a} = [a_0 \ a_1 \ \cdots \ a_{2N_{CH}-2}]^T$ is a vector that contains $2N_{CH} - 1$ coefficients of $a(n)$, once it is the impulse response that results from the convolution between $h_{1R}(n)$ and $h_{R1}(n)$, with both having length N_{CH}, and $\mathbf{b} = [b_0 \ b_1 \ \cdots \ b_{2N_{CH}-2}]^T$, is the vector that contains the $2N_{CH} - 1$ coefficients of $b(n)$ for the same reasons. Furthermore, let $\mathbf{w} = [w_0 \ w_1 \ \cdots \ w_{N-1}]^T$ be a vector containing N samples of $w(n)$, and $\mathbf{y} = [y_{1,0} \ y_{1,1} \ \cdots \ y_{1,N+2(N_{CH}-1)}]^T$ be a vector of N samples from the received signal at node 1, where $y_{i,n}$ represents the received sample at instant n at node i.

Then, it is possible to write (3) in the following matrix form as:

$$\mathbf{y} = \mathbf{M}\mathbf{h} + \mathbf{w}. \quad (8)$$

The least squares estimation of $\mathbf{h}$ can be obtained from [12]

$$\hat{\mathbf{h}} = \arg\min_{\mathbf{h}} |\mathbf{y} - \mathbf{M}\mathbf{h}|^2, \quad (9)$$

whose solution is given by

$$\hat{\mathbf{h}} = \mathbf{M}^{\dagger}\mathbf{y}, \quad (10)$$

where $\mathbf{M}^{\dagger}$ denotes the pseudoinverse matrix of $\mathbf{M}$ and is given by

$$\mathbf{M}^{\dagger} = (\mathbf{M}^H\mathbf{M})^{-1}\mathbf{M}^H. \quad (11)$$

and $\hat{\mathbf{h}}$ contains the estimates $\mathbf{a}$ and $\mathbf{b}$. It is worth noting that the estimated cascaded channels obtained from (10) will be used for self-extraction and equalization at the end nodes.

It is also important to highlight that, although the estimator is based in classic equations ((9) and (10)), some adaptations in the model were necessary to employ it with ANC system.

IV. Simulation Results and Discussion

To evaluate the performance of the proposed LS channel estimator, an ANC system is simulated using binary phase-shift keying (BPSK) modulation.

The gain α is set in a way that the power of the transmitted signal at the relay is equal to the power at nodes 1 and 2, i. e., $P_R = P_1 = P_2$, so that the signal-to-noise ratio (SNR) is the same at all channels. This can be done by considering channels

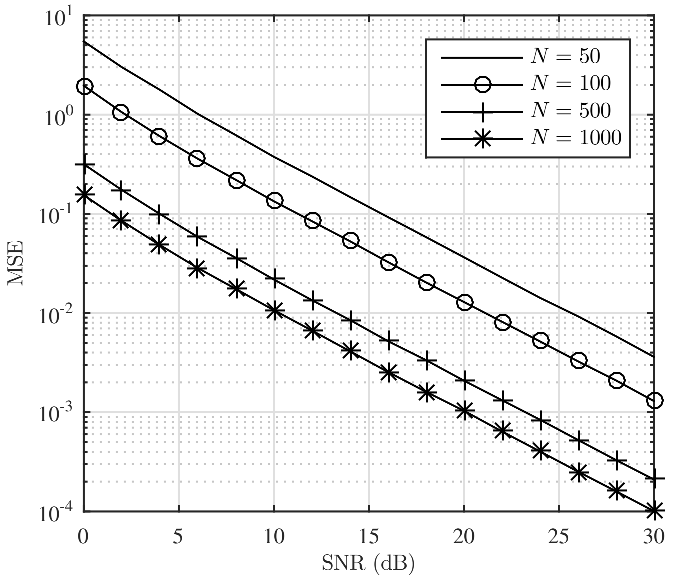

Fig. 2. MSE of $\hat{\mathbf{a}}$ for different training sequence lenghts.

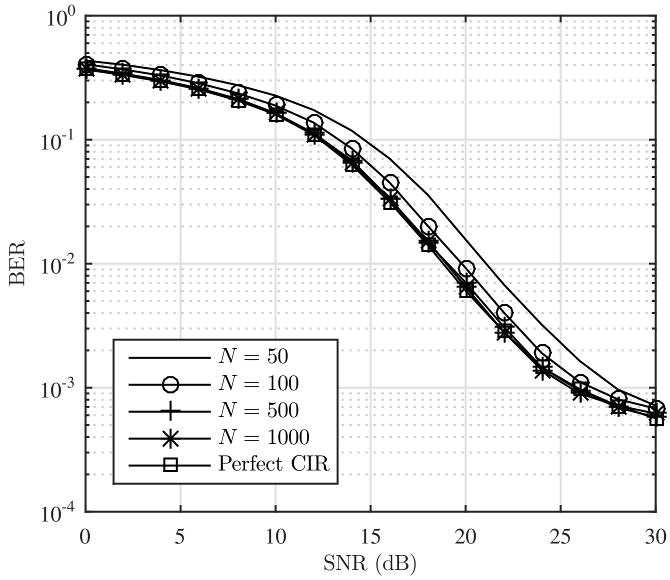

Fig. 3. BER for different training sequence lengths at the LS estimator.

with unity average power. Then, the SNR can be defined as $\mathrm{SNR} = P_R/\sigma_W^2$, where σ_W^2 is the noise power.

The mean squared error (MSE) between the original and estimated channels $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$, and the bit error rate (BER) when the system uses the estimated CIR and when it has the perfect knowledge of the CIR are computed through Monte Carlo simulations for different scenarios. Each realization simulates the transmission of 10^3 information bits. For each SNR, a total of 10^4 realizations are performed for averaging. The channels have lengths $N_{CH} = 5$ and their coefficients are generated randomly in each realization, each one with a zero mean and unity variance Gaussian distribution. The training sequence, that consists of N random BPSK symbols, is concatenated in the beginning of the block of information bits. The estimate $\hat{\mathbf{a}}$ is used to compute the self-extraction operation shown in (5). Then, the estimate $\hat{\mathbf{b}}$ is used to perform the equalization through an MLSE equalizer. Both estimates were obtained from (10). The estimation MSE is computed in each realization by (12), and it is averaged by the number of realizations. A similar expression can be used to compute the MSE for channel $\hat{\mathbf{b}}$.

$$\mathrm{MSE}(\hat{\mathbf{a}}) = \frac{1}{2N_{CH}-1} \sum_{n=0}^{2N_{CH}-2} |a_n - \hat{a}_n|^2, \qquad (12)$$

In the first scenario the MSE and BER performances of the proposed LS estimator are evaluated for different lengths of the training sequence. Figure 2 shows the MSE performance. As the MSE for both concatenated channels are equal, only the MSE for channel $\hat{\mathbf{a}}$ is shown.

As expected, the estimator performance degrades as the length of the training sequence decreases. For an SNR of 10 dB, the training sequence of length $N = 50$ has an MSE performance of only 0.378, while for a length of $N = 100$ it is 0.132. For $N = 500$, the MSE is 0.022, and for $N = 1000$ it achieves an MSE of 0.011.

In Figure 3 is shown the BER for different lengths of training sequence. As observed in Figures 2 and 3, although the MSE is better for a higher training sequence length, it does not always improve significantly the BER. For instance, for $N = 500$, $N = 1000$ and perfect CIR, the BER are practically the same, and the differences can be due to errors in numerical simulation. For $N = 50$, $N = 100$ and $N = 500$, it is possible to see an improvement in the BER. However, using $N = 500$ instead of $N = 100$ saves less than 1 dB in SNR to achieve almost the same BER. For an MSE of 10^{-2} dB there is a difference of approximately 12 dB between the PNC system that uses a $N = 1000$ (the length of a block) and the PNC system that uses $N = 100$, i. e. 10 % of the block length. However, this difference does not impact significantly the BER. Therefore, the proposed estimator can be deployed without the need of a high number of training symbols.

The second scenario evaluates the impact of erroneous channel length L at the estimator, i.e., when it considers a smaller or greater channel length than the actual channel length N_{CH}. For this simulation was used a training sequence of length $N = 100$. Figure 4 shows that the BER does not change significantly when $L > N_{CH}$, once the extra coefficients given by the estimator are nearly zero, but decreases considerably when $L < N_{CH}$. For $L = 3$, there are almost no reduction in BER as the SNR increases. This happens because the intersymbol interference (ISI) generated by the frequency selectivity of the wireless channel are not totally mitigated. First, the self removal given by (5) are not performed correctly because the estimate $\hat{\mathbf{a}}$ does not consider all the coefficients. Second, as the equalizer uses the information $\hat{\mathbf{b}}$, that was obtained considering less coefficients then the actual cascaded channel, its output will have residual ISI, which degrades the overall performance of the system.

Finally, the last scenario compares the proposed estimation technique with the LS estimator proposed in [7] for OFDM-based PNC. Although it is an ANC based on OFDM modulation, the channel estimator estimates the CIR after removing

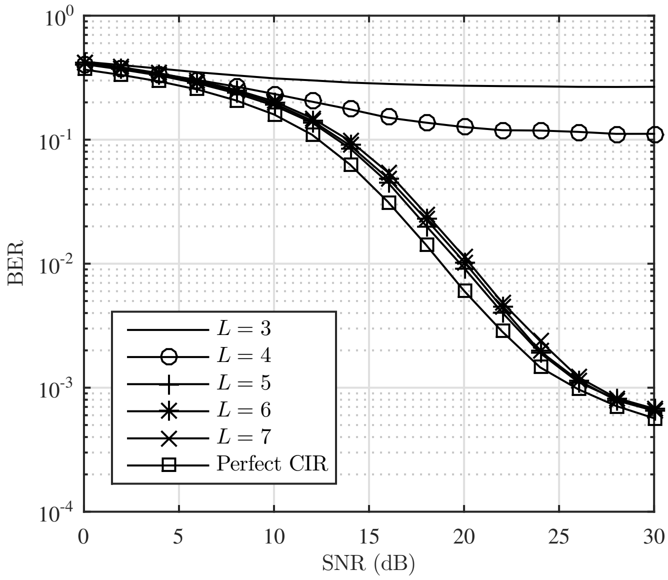

Fig. 4. BER considering different channel lengths at the estimator.

the Cyclic Prefix (CP) and before computing the Discrete Fourier Transform, i. e., it works in the time domain, making it a fair comparison for the MSE. The OFDM system uses 64 carriers and CP of length 16. In this scenario, the proposed technique uses $N = 64$ training symbols to match with the length of the sequence used for the OFDM system, once it uses one OFDM block as training sequence, i. e., 64 training symbols.

It can be seen from Figure 5 that the performance of both estimator are equivalent, and minor differences are due to numerical errors in simulation, although they are different estimators, once the construction of the matrix given by (6) is different, since this estimator deals with SC modulation, and the one in [7] deals with OFDM.

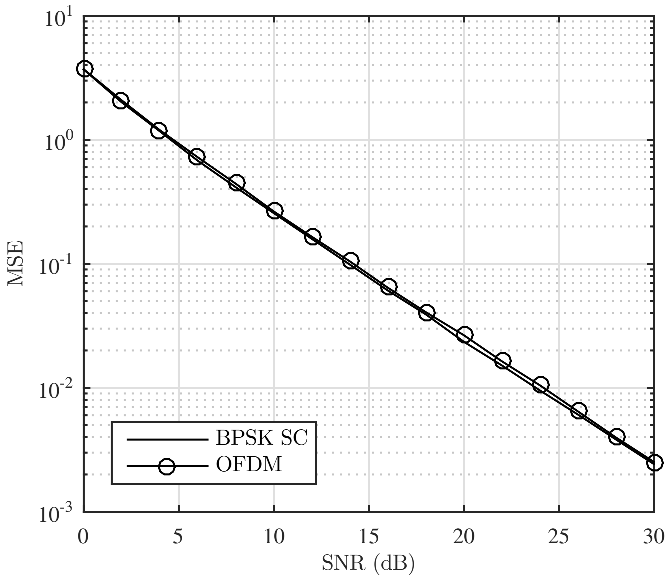

Fig. 5. MSE comparison between the LS estimator for OFDM system and LS estimator for SC system.

V. Conclusions

In this paper a Least Squares channel estimation technique is proposed for PNC communication systems that uses single carrier modulation and operates under frequency-selective fading channels.

It is shown that a higher training sequence length performs better in terms of MSE. However, it is possible to see that a high accurate estimation does not necessary imply in a better BER performance, and, therefore, it can be used shorter training sequences and still obtain nearly the optimal performance.

Furthermore, tests considering a different channel length at the estimator than the real one shows that using a higher channel length at the estimator does not bring penalty when compared to the real length, keeping an equivalent BER performance. On the other hand, considering a channel length smaller than the real one degrades the BER performance considerably, hence, it is important the development of accurate estimator for the channel length at the end nodes.

The proposed estimator was also compared to the estimator presented in [7]. Although this last one is an OFDM system and have different mathematical constructions, the estimator works in time domain, so the MSE comparison is fair. It is shown that both estimators have equivalent MSE performance.

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