# Performance Evaluation of Spatial Filtering for Relay-Assisted MIMO Communication Systems

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***Resumo*— Neste artigo, consideramos um sistema de comunicação sem fio MIMO (do inglês, *Multiple-Input Multiple-Output*) dotado de um repetidor, o que torna possível o estabelecimento da comunicação fonte-destino. Para tal sistema, avaliamos a taxa de erro de bits tomando três tipos de filtragem espacial e as combinando na fonte e no repetidor. O intuito deste trabalho é identificar a melhor abordagem para uma dada configuração de sistema e relação sinal ruído.**

***Palavras-Chave*— MIMO, filtragem espacial, repetidor, taxa de erro de bits.**

***Abstract*— We consider a dual-hop single-relayed Multiple Input Multiple Output (MIMO) wireless communication system containing a relay that enables a link among source and destination. For this system, we evaluate the bit error rate for three different combinations of spatial filtering at the source and relay. The goal of this work is to identify the best approach for a given system configuration and signal to noise ratio.**

***Keywords*— MIMO, spatial filtering, relay, bit error rate.**

## I. INTRODUCTION

The fast growing of access to wireless communications requires a higher quantity of available frequency bands that, sometimes, is impossible due to spectral scarcity. A solution to this problem is to increase spectral efficiency of communication systems in their designs. Multiple Input Multiple Output (MIMO) techniques, which exploits multiple antennas at both the transmitter and receiver, have been drawing considerable attention due to the unprecedented capacity increase they can provide. By Using these techniques in a communication system, we intend to provide significant improvement in the spectral efficiency and link reliability due to multiplexing and diversity gains [1],[2].

Relaying techniques have appeared recently as viable options for challenging the trade-off between transmission range and end-to-end data rate [3]. Applying relays in a cooperative communication system, not only efficiently extends coverage and eliminates dead spots, but also considerably provides extra spatial diversity gains. By combining MIMO techniques with relaying ones allows to simultaneously increase data rate and coverage [1]. For further details see, e.g. [3]-[6]. Two relay processing techniques commonly used in MIMO relayed systems are amplifying-and-forward (AF) and decode-and-forward (DF). In the first one, the relay simply amplifies the received signal before transmission to the destination, in order to combat path loss and noise effects. In the second one, the relay demodulates and decodes the signal prior to re-encoding and retransmission. We focus on amplify-and-forward technique due to its simplicity.

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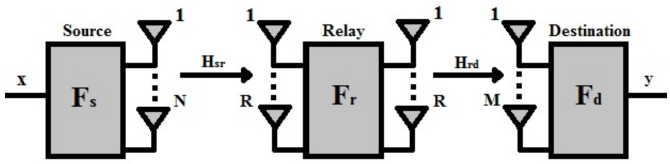


Fig. 1. Dual-hop single-relayed MIMO communication system.

By exploiting channel knowledge at transmitter/receiver in both source-to-relay (*S-R*) and relay-to-destination (*R-D*) links (which means that the channel matrices $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$ are assumed to be known), we evaluate the performance of three combined transmit-receive beamforming techniques in a single-relayed MIMO communication system. We consider Zero Forcing (ZF), Minimum Mean Square Error (MMSE) and Singular Value Decomposition (SVD) solutions, which are the most popular in this context. The goal of this study is to conclude about the techniques for a fixed system configuration.

## II. SYSTEM MODEL

Figure 1 depicts a dual-hop single-relayed MIMO wireless communication system with $N$ antennas at source, $R$ antennas at relay and $M$ antennas at destination. We consider a half-duplex non-regenerative relaying scenario, where it takes two non-overlapping time-slots for the data to be transmitted from source to destination via *S-R* and *R-D* with MIMO channel matrices $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$, respectively. We assume deep large-scale fading effect due to the long distance source/destination, which disables a direct link. Another assuming is that channel matrices are completely known by all system devices (i.e. source, relay and destination).

In the *S-R* time slot, the source broadcasts the data signal $\mathbf{x} \in \mathbb{C}^{N\times 1}$ through *S-R* channel after a linear processing using the source spatial filter $\mathbf{F}_s \in \mathbb{C}^{N\times N}$. The corresponding received signal vector at the relay is written as:

$$\mathbf{r} = \mathbf{H}_{sr}\mathbf{F}_s\mathbf{x} + \mathbf{n}_r \in \mathbb{C}^{R\times 1} \quad (1)$$

where $\mathbf{n}_r$ is the additive noise vector, whose entries are modeled as zero-mean complex Gaussian random variables with variance $\sigma_r^2$. After receiving the signal, the relay processes its received data signal by applying a spatial filter $\mathbf{F}_r \in \mathbb{C}^{R\times R}$ yielding:

$$\mathbf{t} = \mathbf{F}_r\mathbf{r} \in \mathbb{C}^{R\times 1}, \quad (2)$$

assuming that the destination device employs a spatial filter $\mathbf{F}_r \in \mathbb{C}^{M\times M}$, the signal vector at its input is given by:

$$\mathbf{y} = \mathbf{F}_d(\mathbf{H}_{rd}\mathbf{t} + \mathbf{n}_d) \in \mathbb{C}^{N\times 1}, \quad (3)$$

where $\mathbf{n}_d$ and $\sigma_d^2$ have the same characteristics of $n_r$ and $\sigma_r^2$. Combining (1), (2) and (3):

$$\begin{aligned} \mathbf{y} &= \mathbf{F}_d(\mathbf{H}_{rd}\mathbf{F}_r(\mathbf{H}_{sr}\mathbf{F}_s\mathbf{x} + \mathbf{n}_r) + \mathbf{n}_d) \\ &= \mathbf{F}_d\mathbf{H}_{rd}\mathbf{F}_r\mathbf{H}_{sr}\mathbf{F}_s\mathbf{x} + \mathbf{n}'_d, \end{aligned} \quad (4)$$

where $\mathbf{n}'_d = \mathbf{F}_d(\mathbf{H}_{rd}\mathbf{F}_r\mathbf{n}_r + \mathbf{n}_d)$ denotes the total noise component at the destination.

## III. Spatial Filtering Techniques

In the following, we briefly recall three spatial filtering techniques that are used to design the matrix filters $\mathbf{F}_s$, $\mathbf{F}_r$ and $\mathbf{F}_d$, namely, the Zero Forcing (ZF), Minimum Mean Square Error (MMSE) and Singular Value Decomposition (SVD).

We separated the filtering processes among the participant devices, so, we have a pre-filter in the transmitter device and a post-filter in the receiver device for each time-slot. To adapt the system model for this change we separated the relay filter matrix $\mathbf{F}_r$:

$$\mathbf{F}_r = \mathbf{F}_{rd_{PrF}}\mathbf{F}_{sr_{PoF}} \quad (5)$$

where $\mathbf{F}_{rd_{PrF}}$ is the pre-filter matrix of the *R-D* time-slot and $\mathbf{F}_{sr_{PoF}}$ is the post-filter matrix of the *S-R* time-slot. We applied this to enable the application of the SVD technique, but it proved its efficiency in another results, however, the relay have to manage a higher computational cost.

The ZF technique consists in nulling out the channel effects making the filter act as the pseudo-inverse of the channel matrix. The pre-filtering and post-filtering matrices are written as

$$\mathbf{F}_{ZF_{PrF}} = \mathbf{H}^H, A_{Rc} \geq L, \quad (6)$$

$$\mathbf{F}_{ZF_{PoF}} = (\mathbf{H}\mathbf{H}^H)^{-1}, A_{Rc} \geq L, \quad (7)$$

where $A_X$ is a general parameter and represents the number of antennas in the device that has the $X$ function which can be transmit $(Tr)$ or receive $(Rc)$. $L$ is the rank of the channel matrix. This technique also affects the noise in the receive antennas, in order that, if the noise is improved by this method, we can also have a lot of error, but it still a good solution.

The MMSE technique consists in nulling out the channel effects in the same way of ZF technique, but it takes into count the noise effects and compensates it. The pre-filtering and post-filtering matrices are written as

$$\mathbf{F}_{MMSE_{PrF}} = \mathbf{H}^H, A_{Rc} \geq L, \quad (8)$$

$$\mathbf{F}_{MMSE_{PoF}} = (\mathbf{H}\mathbf{H}^H + \sigma_{Rc}^2\mathbf{I}_{Rc})^{-1}, A_{Rc} \geq L, \quad (9)$$

where $\sigma_{Rc}^2$ is the noise variance in the receiver device.

The SVD technique partitions the channel matrix into two orthonormal matrices, and among them, a eigenvalue matrix as showed in (10). The pre-filtering and post-filtering matrices are written as

$$\mathbf{H} = \mathbf{U}\mathbf{S}\mathbf{V}^H; L \leq min(A_{Tr}, A_{Rc}) \quad (10)$$

$$\mathbf{F}_{SVD_{PrF}} = \mathbf{V}, \mathbf{F}_{SVD_{PoF}} = \mathbf{S}^{\dagger}\mathbf{U}^H; Rc = Destination \quad (11)$$

or,

$$\mathbf{F}_{SVD_{PrF}} = \mathbf{V}\mathbf{S}^{\dagger}, \mathbf{F}_{SVD_{PoF}} = \mathbf{U}^H; Rc = Relay, \quad (12)$$

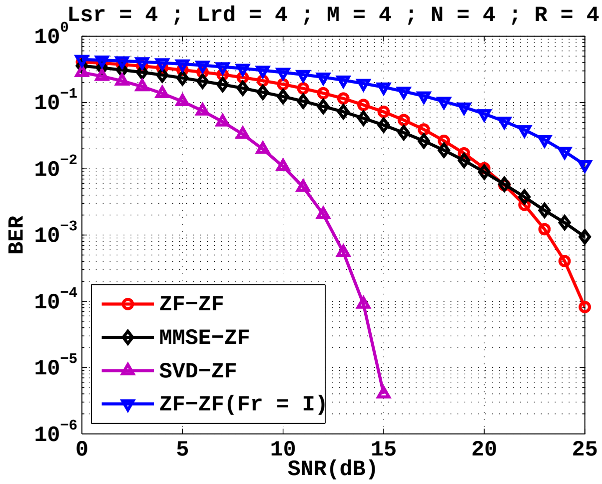


Fig. 2. Performance of three spatial filtering solutions for the S-R link.

where $\mathbf{U}$ and $\mathbf{V}$ are the orthonormal matrices and $\mathbf{S}$ is the eigenvalue matrix. By placing these filters in a time-slot, due to the orthonormality of $\mathbf{U}$ and $\mathbf{V}$, they form identities around $\mathbf{S}$, thereby, we engage its pseudo-inverse matrix $\mathbf{S}^{\dagger}$ nulling out channel effects.

## IV. Simulation Results and Discussion

We used Gaussian distributions to generate the channel matrices and the noise vectors. The transmitted symbols are BPSK modulated. We have performed several simulations to evaluate the performance of the different spatial filtering solutions as functions of the channel matrix rank and the number of antennas. In this paper, we have selected to show the result of Figure 2, which depicts BER vs. SNR curves for three spatial filtering solution pairs (indicated as 'X-Y' in the figure) and one which we exempted the relay of computational cost to show the gain provided by dividing the filters in pre-filter and post-filter. The ZF filter is fixed at the second time slot. We can see that, for higher SNRs, ZF-ZF is a better solution than MMSE-ZF as a technique for a full rank channel matrix, while MMSE-ZF one is a better solution at lower SNRs, as expected. The improvement of dividing the filter is verified when we compare ZF-ZF solution curves, we can see at $10^{-2}$ BER value that we have a 5 dB gain. The best solution turns out to be SVD-ZF solution which curve has the most accentuated decay.

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