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Transmit Beamforming in Relayed Multicast Systems

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Abstract— **In this work the gains from the addition of a single antenna relay in a multicasting system with transmitter beamforming are studied. Different solutions to the problem of maximizing the minimum user SNR with varying trade-offs between performance and complexity are taken from the literature and studied in two relaying scenarios. One scheme represents the classic setup without using the relay while the other combines the signal from the direct and relayed paths using Maximal Ratio Combining (MRC). The gain in received signal power from the relayed scheme comes at the expense of an additional transmission phase. The simulation results show gains on minimal SNR and those gains resulted in superior spectral efficiency and throughput even when using double as much transmission phases.**

Keywords— **precoding, relay, multicasting, SNR maximization**

I. INTRODUCTION

With the growth on the demand for different data services over wireless networks, the deployment of multicast and broadcast services on cellular networks is a relevant topic of interest.

Services like streaming may use the system resources more efficiently by delivering the data with the same resource to a number of User Equipments (UEs) in a multicast scenario, instead of using multiple unicast transmissions of the same data. In the following, we present a brief overview of the state of the art on multicast beamforming for maximizing the minimum Signal-to-Noise Ratio (SNR) among UEs.

In [1], the authors consider the transmitter beamforming in a multicast system to minimize the total transmission power or maximize the minimal UE SNR. Those problems were proven to be NP-hard and solved using Semi Definite Relaxation (SDR). A different approach using an efficient suboptimal algorithm was proposed in [2]. Different beamforming techniques were applied to multicast and unicast systems with the objective of maximizing the minimal UE SNR. The proposed solution particularly benefits from scenarios with strong line-of-sight. An iterative algorithm, which iteratively improves the worst-UE SNR and has a complexity lower than that of SDR was proposed in [3]. The authors show that, for large group sizes, this iterative algorithm achieves better results than the SDR approach.

As for relaying techniques, they are suitable to increase signal strength and coverage for wireless networks. In a cellular environment, a relay can be deployed in areas where there are strong shadowing effects, such as inside buildings and tunnels. In the following some recent works dealing with relaying systems are presented.

The work in [4] deals with a unicast system where all nodes have multiple antennas and apply beamforming to maximize the sum-rate. That work assumes an Amplify and Forward (AF) relay and provides the optimal sum-rate solution to the scenario without direct links between the transmitter and the receivers. When a direct link is present, upper and lower bounds of the optimal system capacity are discussed. Also restricted to a unicast system, the authors of [5] employ space-time diversity using two relays to increase the achievable rate in a unicast system with multiple antennas at each node. The performance evaluation shows that the proposed system can provide significant gains over the conventional point-to-point and half-duplex relay scenarios.

As for relaying in multicast systems, such a scenario is considered in [6]. The authors employ network coding to optimally maximize throughput. It also considers delay and queue length constraints. While both the direct and relayed links are considered, only single antenna nodes are used. Relayed beamforming in multicast systems is considered in [7], but ignoring the direct link between transmitter and receivers. The authors design a computationally efficient beamforming scheme to minimize the total transmitted power at the relays subject to quality-of-service constraints. Simulation results showed that their technique outperforms the SDR-based technique.

None of these previous works, however, has approached the problem of beamforming in a multicast system to maximize the minimum UE SNR using a single relay and considering both direct and relayed links. The main contribution of this paper is therefore to propose a relayed multicast beamforming scheme that takes advantage of spatial diversity in a two-phase transmission, which is shown to provide gains in terms of SNR and throughput with regard to the case without relaying.

This paper is organized as follows. In section II the general two-phase relayed multicast scenario is presented. In section III, we define the general problem of maximizing the minimal UE SNR in a cell with one relay. In section IV the relaying problem is presented, while the precoding problem is presented in section V. Simulation results are shown in section VI and conclusions are drawn in section VII.

Notation: Vectors and matrices are denoted by bold-faced lower and upper case italic letters, respectively. The operators $()^T$, $()^H$, and $()^*$ stand for transposition, Hermitian transposition, and complex conjugate, respectively. tr() denotes the trace of a matrix and $\angle()$ denotes the angle of a complex scalar. $\mathbf{0}$ denotes a zero valued vector, $\mathbf{1}$ a one valued vector and $\boldsymbol{U}$ a one valued square matrix with suitable dimensions.

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II. SYSTEM MODELING

We assume a two-phase multicast relay-aided scenario with a group of J single-antenna UEs and one single-antenna AF relay distributed on one cell.

In the first phase, the signal x is transmitted to all UEs and one relay from A antennas in the Base Station (BS) weighted by the precoder vector $\boldsymbol{m} \in \mathbb{C}^{A\times 1}$, with $\|\boldsymbol{m}\| = 1$.

The signal received at the UE j in the first phase $y_{1,j}$ is given by

$$y_{1,j} = \boldsymbol{h}_{1,j}^T \boldsymbol{m} x + n_{1,j}, \tag{1}$$

where $\boldsymbol{h}_{1,j} \in \mathbb{C}^{A\times 1}$ is the channel from the BS to the UE j in the first transmission phase and $n_{1,j}$ is the Additive White Gaussian Noise (AWGN) at UE j during the first phase.

The SNR at the UE j during the first phase is given by

$$\gamma_{1,j} = \frac{E[|\boldsymbol{h}_{1,j}^T \boldsymbol{m} x|^2]}{E[|n_{1,j}|^2]} = \frac{|\boldsymbol{h}_{1,j}^T \boldsymbol{m}|^2 p_x}{\sigma^2}, \tag{2}$$

where p_x is the transmit signal power and σ^2 is the AWGN variance. The signal received at the relay y_r is given by

$$y_r = \boldsymbol{h}_2^T \boldsymbol{m} x + n_r, \tag{3}$$

where $\boldsymbol{h}_2 \in \mathbb{C}^{A\times 1}$ is the channel from the BS to the relay and n_r is the AWGN at the relay.

During the second phase, only the relay transmits to the UEs. The signal received at the UE j from the relay during the second phase $y_{2,j}$ is given by

$$y_{2,j} = d_r h_{3,j} y_r + n_{2,j} = d_r h_{3,j} \boldsymbol{h}_2^T \boldsymbol{m} x + d_r h_{3,j} n_r + n_{2,j}, \tag{4}$$

where $d_r \in \mathbb{R}$ is the signal gain provided by the AF relay, $h_{3,j} \in \mathbb{C}$ is the channel from the relay to the UE j and $n_{2,j}$ is the AWGN at UE j during the second phase.

The SNR at the UE j during the second phase is given by

$$\gamma_{2,j} = \frac{E[|d_r h_{3,j} \boldsymbol{h}_2^T \boldsymbol{m} x|^2]}{E[|d_r h_{3,j} n_r + n_{2,j}|^2]} = \frac{|d_r h_{3,j} \boldsymbol{h}_2^T \boldsymbol{m}|^2 p_x}{|d_r h_{3,j}|^2 \sigma^2 + \sigma^2}. \tag{5}$$

The maximum value of d_r is limited by the relay transmit power p_r. Using the Cauchy-Schwarz inequality in the signal transmitted by the relay we get that p_r is limited by

$$p_r = E[|d_r y_r|^2] \leq d_r^2 (\|\boldsymbol{h}_2\|^2 \|\boldsymbol{m}\|^2 p_x + \sigma^2), \tag{6}$$

using $\|\boldsymbol{m}\| = 1$ we get that d_r is limited by

$$d_r \leq \sqrt{\frac{p_r}{\|\boldsymbol{h}_2\|^2 p_x + \sigma^2}}. \tag{7}$$

The received signals at the UE j during both phases $y_{1,j}$ and $y_{2,j}$ are combined according to

$$y_j = d_{1,j} y_{1,j} + d_{2,j} y_{2,j}, \quad d_{1,j}, d_{2,j} \in \mathbb{C} \tag{8}$$

leading to the final SNR γ_j given by

$$\gamma_j = \frac{|(d_{1,j} \boldsymbol{h}_{1,j}^T + d_r d_{2,j} h_{3,j} \boldsymbol{h}_2^T) \boldsymbol{m}|^2 p_x}{(|d_{1,j}|^2 + |d_r d_{2,j} h_{3,j}|^2 + |d_{2,j}|^2) \sigma^2}. \tag{9}$$

This setup is presented in Fig. 1, where the solid arrows represent the transmission on the first phase and the dashed arrows represent the second phase.

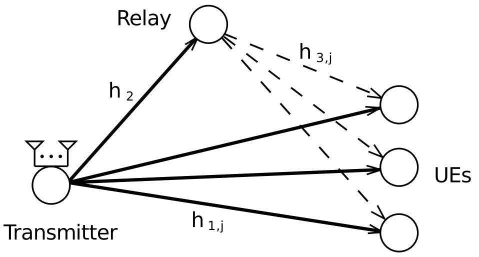

Fig. 1. Two-phase transmission scheme.

III. RELAYED MULTICASTING PROBLEM

In this section we present the general problem studied in this work. We consider the problem of maximizing the minimum SNR among the UEs in the cell by adapting the precoder vector $\boldsymbol{m}$, the first phase receive weight vector $\boldsymbol{d}_1 = [d_{1,1}, \ldots, d_{1,J}]$, the second phase receive weight vector $\boldsymbol{d}_2 = [d_{2,1}, \ldots, d_{2,J}]$ and the signal gain d_r. This problem is presented as follows.

$$\begin{aligned} \max_{\boldsymbol{d}_1, \boldsymbol{d}_2, d_r, \boldsymbol{m}} \ \min_{j} \quad & \gamma_j(\boldsymbol{d}_1, \boldsymbol{d}_2, d_r, \boldsymbol{m}) \\ s.t. \quad & \\ d_r \quad & \leq \sqrt{\frac{p_r}{\|\boldsymbol{h}_2\|^2 p_x + \sigma^2}} \\ \|\boldsymbol{m}\| \quad & = 1 \end{aligned} \tag{10}$$

We propose to solve the problem suboptimally by dividing it into two sub-problems. The *Relaying Problem* and the *Precoding Problem*. Those two problems are discussed in the following sections.

IV. RELAYING PROBLEM

In this section, the relaying problem is presented. In this problem, the precoding vector $\boldsymbol{m}$ is assumed to be already specified. Thus only $\boldsymbol{d}_1$,$\boldsymbol{d}_2$ and d_r must be optimized. This problem is presented below.

$$\begin{aligned} \max_{\boldsymbol{d}_1, \boldsymbol{d}_2, d_r} \ \min_{j} \quad & \gamma_j(\boldsymbol{d}_1, \boldsymbol{d}_2, d_r, \boldsymbol{m}) \\ s.t. \quad & \\ d_r \quad & \leq \sqrt{\frac{p_r}{\|\boldsymbol{h}_2\|^2 p_x + \sigma^2}} \end{aligned} \tag{11}$$

We consider two different transmission schemes for the multicasting scenario. The first scheme does not use the relay at all. This scheme is equivalent to using the weights $\boldsymbol{d}_1 = \mathbf{1}$ and $\boldsymbol{d}_2 = \mathbf{0}$ and the general problem is restricted to the precoding problem of section V.

The other scheme uses the relay by performing Maximal Ratio Combining (MRC) among the two signals received by a UE during the different transmission phases to calculate the values of $\boldsymbol{d}_1$ and $\boldsymbol{d}_2$.

The relaying gain d_r is set to the maximum allowed value. This choice of d_r is suboptimal as d_r amplifies not only the relayed signal but also the noise received at the relay. Thus, this choice is suitable to situations where the power of the

Algorithm 1 Maximal Ratio Combining (MRC) algorithm.

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$d_r \leftarrow \sqrt{\frac{p_r}{\|\boldsymbol{h}_2\|^2 p_x + \sigma^2}}$
$\boldsymbol{m} \leftarrow F(\boldsymbol{h}_1)$
for $j = 1 \dots J$ do
    $d_{1,j} \leftarrow \frac{(\boldsymbol{h}_{1,j}^T \boldsymbol{m})^*}{\sigma^2}$
    $d_{2,j} \leftarrow \frac{(d_r h_{3,j} \boldsymbol{h}_{2,j}^T \boldsymbol{m})^*}{(1 + |d_r h_{3,j}|^2)\sigma^2}$
end for
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signal received by the relay is much larger than the noise power.

This scheme is presented in Algorithm 1, where $F(\boldsymbol{h}_1)$ is one of the precoding algorithms in Section V considering only the direct channel during the first phase as in the previous scheme.

V. PRECODING PROBLEM

In this section, the precoding algorithms considered in this work are presented. In this problem, the relay is ignored and the precoding vector $\boldsymbol{m}$ is optimized to maximize the direct link SNR $\gamma_{1,j}$ as follows.

$$\begin{aligned} \max_{\boldsymbol{m}} \min_{j} & \quad \gamma_{1,j}(\boldsymbol{m}) \\ s.t. & \\ \|\boldsymbol{m}\| &= 1 \end{aligned} \tag{12}$$

This problem is known to be NP-hard [1] and multiple proposed solutions are available in the literature [1]–[3].

A. Matched Filter (MF)

In [8] the MF precoder is extended to a multicast scenario by solving the problem.

$$\begin{aligned} \max_{\boldsymbol{m}} & \quad \frac{|E\{x\boldsymbol{1}^H \boldsymbol{y}_1\}|^2}{E\{\|x\boldsymbol{1}\|^2\}E\{\|\boldsymbol{n}_1\|^2\}} \\ s.t. & \\ \|\boldsymbol{m}\| &= 1 \end{aligned} \tag{13}$$

where $\boldsymbol{n}_1 = [n_{1,1}, \dots, n_{1,j}, \dots, n_{1,J}]$. Through Lagrange optimization the optimum $\boldsymbol{m}$ is given by

$$\boldsymbol{m} = \sqrt{\frac{p_x}{\sigma^2 \text{tr}(\boldsymbol{H}\boldsymbol{H}^H \boldsymbol{U})}} \boldsymbol{H}^H \boldsymbol{1}, \tag{14}$$

where $\boldsymbol{H} = [\boldsymbol{h}_1^T, \dots, \boldsymbol{h}_j^T, \dots, \boldsymbol{h}_J^T]^T$.

B. User-Selective Matched Filter (USMF)

In [2] and [8] an efficient heuristic algorithm to solve the precoding problem was proposed. The USMF aims at achieving a trade-off between the provision of user fairness and the low complexity of the matched filter.

This algorithms tries to solve the problem by choosing an $\boldsymbol{m}$ that is the sum of the MFs of a subset of the system UEs as given by

$$\boldsymbol{m} = \sqrt{\frac{p_x}{\sigma^2 tr(\boldsymbol{C}^T \boldsymbol{H}\boldsymbol{H}^H \boldsymbol{C}\boldsymbol{U})}} \boldsymbol{H}^H \boldsymbol{C}\boldsymbol{1}, \tag{15}$$

where $\boldsymbol{C} \in \mathbb{Z}^{J \times J}$ is a diagonal matrix with elements $c_{j,j} \in \{0, 1\}$ that assumes value one when the matched filter to UE j is selected and zero otherwise. Note that the MF algorithm of the previous section corresponds to a special case of USMF when $\boldsymbol{C} = \boldsymbol{I}$.

Since there are J UEs, and the diagonal elements of $\boldsymbol{C}$ are restricted to binary values, there exists a total of $2^J - 1$ possible matrices. Due to this exponential complexity, a correlation-based algorithm for determining $\boldsymbol{C}$ was proposed in [2].

Let $\rho_{i,j}$ denote the correlation between the vector channels of UEs i and j, which is given by the normalized scalar product [9]

$$\rho_{i,j} = \frac{|\boldsymbol{h}_i \boldsymbol{h}_j^H|}{\|\boldsymbol{h}_i\| \|\boldsymbol{h}_j\|}, \tag{16}$$

for which $\rho_{i,i} = 1$ and $\rho_{i,j} = \rho_{j,i}$.

All pairs of channels are sorted in their decreasing order of correlation and it is assumed initially that $\boldsymbol{C} = \boldsymbol{I}$.

For each pair of channels $\{i, j\}$, we calculate the gain to the highest minimum SNR Δ_1 of setting $c_{i,i} = 0$ and the gain Δ_2 of setting $c_{j,j} = 0$.

If $\Delta_1 < 0$ and $\Delta_2 < 0$, nothing is done. In the opposite case, we set $c_{i,i} = 0$ if $\Delta_1 > \Delta_2$ or $c_{j,j} = 0$ otherwise.

C. Semi Definite Relaxation (SDR)

A more efficient suboptimal solution, but of higher computational complexity, to the maximization of the minimum SNR problem was proposed in [1] based on Semi Definite Relaxation (SDR). The optimization problem is rewritten in an equivalent form, in which the non-convex term is expressed by a rank-one constraint.

The idea is to drop the rank-one constraint and solve the problem through Semi-Definite Programming, for which there exist very efficient numerical methods, such as those implemented by the SeDuMi Matlab toolbox [10].

If the obtained solution has in fact rank one, then the optimal solution has been achieved and is given by the principal eigenvector of the solution, otherwise the *RandB* randomization method must be employed to approximate the optimal solution.

The *RandB* method assumes that each element m_l of vector $\boldsymbol{m}$ is given by $m_l = \sqrt{\boldsymbol{X}_{l,l}} e^{j\theta}$, where θ is uniformly distributed within $[0, 2\pi]$.

D. Iterative SNR-increasing Update Algorithm (ISUA)

In [3] a solution to the precoding problem was proposed using the heuristic Reduce Complexity Combine-2 algorithm to find a good $\boldsymbol{m}$ and iteratively improve $\boldsymbol{m}$ through the ISUA algorithm.

The Reduce Complexity Combine-2 algorithm calculates the UE l with the lowest channel norm and calculates the two-UE

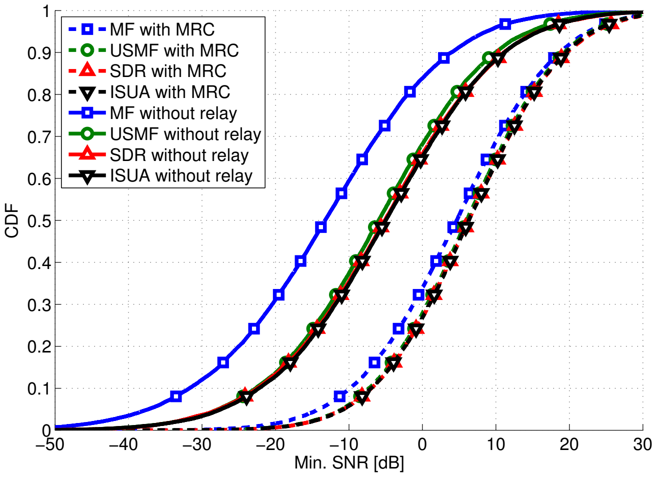

Fig. 2. Minimum UE SNR CDF (fixed load of 4 UEs).

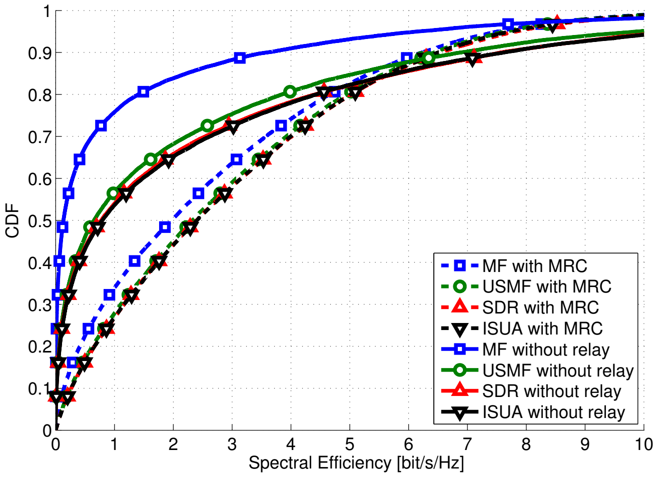

Fig. 3. UE Spectral Efficiency CDF (fixed load of 4 UEs).

optimum $\boldsymbol{m}_i$ pairing the UE l with all other UEs i. Finally, the two-UE optimum $\boldsymbol{m}_i$ with the lowest norm is chosen.

The ISUA algorithm iteratively "rotates" the complex vector $\boldsymbol{m}$. This is achieved by performing a linear combination between the current $\boldsymbol{m}$ and part of the channel of the UE with the lowest SNR that is orthogonal to $\boldsymbol{m}$. If the lowest SNR is reduced after one iteration, the iteration is redone with a lower step α until the SNR increases.

VI. CASE STUDY

The simulation scenario consists of a single hexagonal sector with a 334 m diameter. It is comprised by a base station equipped with a four-element uniform linear antenna array at one of the hexagon edges, a single antenna relay at the center of the sector and several single antenna UEs uniformly distributed on the sector.

We implemented a channel model with no line-of-sight where the fast fading is modeled by zero mean circularly symmetric complex Gaussian random variables with unit variance, a log-normal shadowing with 8 dB variance, path-loss given by

$$L(d) = -35.3 - 37.6 \log_{10}(d) \quad dB \tag{17}$$

and the antenna pattern from [11] given by

$$A(\phi) = -\min(12(\phi/70)^2, 20) \quad dB. \tag{18}$$

The base station power is calculated to ensure a minimum SNR of 10 dB at the sector border considering only path-loss. The maximum relay power is the same as that of the base station and the noise power is of roughly -112 dBm. For each algorithm, 30,000 snapshots were simulated.

In Fig. 2 we present the Cumulative Distribution Function (CDF) of the minimum UE SNR for the simulated algorithms in both schemes, assuming a fixed load of 4 UEs. We can see a gain in minimum SNR for all algorithms when using the MRC scheme. This gain in SNR comes from the additional power provided by the signal amplified and transmitted by the relay. The gain in SNR at the 50^{th} percentile is of more than 15 dB for the MF algorithm and more than 10 dB for the USMF, SDR and ISUA. In both schemes, the SDR and ISUA algorithms present the best performance as a result from their ability to achieve a close to optimal solution to the precoding problem. The USMF is the third best in performance for both schemes confirming its good trade-off between performance and complexity. Finally, the MF algorithm shows the worst performance in both schemes, a result from an objective not directly aimed at the maximization of the minimal SNR. The schemes differ in the difference in the performance of the simulated algorithms. While the difference between the best and worst algorithms at the 50^{th} percentile is more than 7 dB in the scheme without relay, the difference in the MRC scheme is of less than 4 dB.

In Fig. 3, we present the CDF of the UE spectral efficiency obtained using the Shannon capacity formula, normalized by the number of required transmission phases, for all considered precoding and relaying schemes. For each relaying algorithm, the same relative performance from Fig. 2 is observed among the precoding algorithms.

The MF in the scheme without relay presents the worst spectral efficiency, due to the low achieved minimal SNR. Nevertheless, the other precoding algorithms with the scheme without relay achieve a high spectral efficiency at the upper percentiles (above 80%). This is a result from the necessity of a new transmission phase for the relay in the other two schemes. Thus, the scheme without relay is outperformed by the other scheme only when the gain in terms of minimal SNR from the relay outmatches the burden of an additional transmission phase.

In Fig. 4 we present the mean minimum UE SNR with different loads for the simulated algorithms in both schemes. When the number of UEs is lower than the number of antennas (< 4) we see a different relative performance of the simulated algorithms. The SDR presents the worst performance, followed by the MF and the USMF and ISUA are tied as the best. The bad performance of SDR can only be attributed to a higher incidence of solutions with rank > 1 for this low dimension channel matrix and higher dependency on the randomization. The performance at this low load depends only

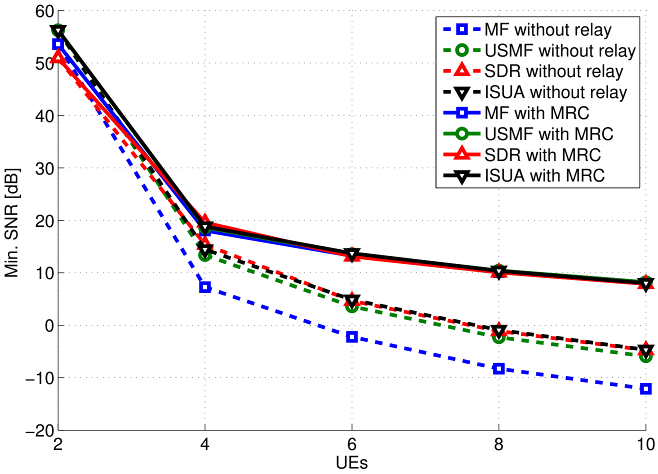

Fig. 4. Average of the Minimum UE SNR per load.

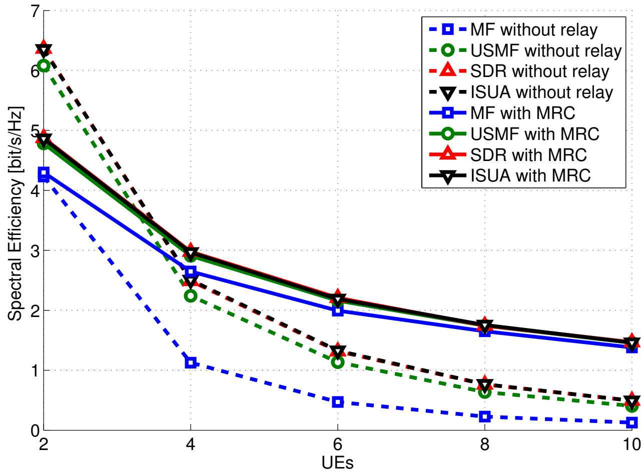

Fig. 5. Average UE Spectral Efficiency per load.

on the algorithm and the same performance is observed for both schemes.

At the higher loads the difference in performance between both schemes becomes higher, with the performance of the scheme without relay decreasing much faster than the MRC. In addition, the difference in performance among the algorithms in the MRC scheme decreases for higher loads, presenting equal performance for all algorithms.

In Fig. 5, we present the average spectral efficiency with different loads for all relaying and precoding schemes. At the load of 2 UEs, the best performance is presented by the SDR, ISUA and USMF algorithms, respectively, when combined with the scheme without relay. This happens due to the additional throughput that comes from using only one transmission phase, which compensates the lower SNR. As the load increases, the performance decreases as a result from the decrease in terms of minimum SNR. Note that the performance of the scheme without relay falls faster than the MRC scheme. Up from 3 UEs, the relayed scheme presents the best spectral efficiency, with all precoding algorithms reaching a similar performance, except for MF, which is slightly worse.

VII. CONCLUSIONS

In this work the gains from the addition of a single antenna relay in a multicasting system with transmitter beamforming were studied. The problem was divided into separate relaying and precoding problems. This decoupling allowed the use of precoding algorithms available in the literature to solve the problem of maximizing the minimum UE SNR in the relayed system without specific changes.

A relaying scheme has been proposed, which takes advantage of the additional link to improve the minimum SNR among the users. In this scheme, the precoding is done based only on the direct link and MRC is additionally performed at the UEs.

Next, simulations were performed and the proposed relaying scheme was shown to present gains in terms of minimal SNR and spectral efficiency, in spite of requiring the double of transmission phases.

In conclusion, the use of relays in the studied scenario is promising and the potential for performance gains is higher in systems with low SNRs and a large number of UEs. As perspective of further studies we can mention the joint optimization and design of the multicast precoding vector and the MRC weights, which might lead to further performance gains.

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