Performance Analysis of a 64-QAM Rotated Constellation Subject to Rice Fading

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***Abstract*— Fading, caused by multipath in wireless communication, can degrade the performance of a digital communication system. A few techniques have been proposed to improve that performance, including constellation rotation. The rotated constellation is a technique that introduces redundancy through a careful choice of the reference constellation angle. This letter presents an evaluation of that technique considering the transmission over a Rician fading channel, which is flexible enough to represent different transmission environments (open, suburban, low-density urban, medium-density urban area). The optimum angle of rotation of a 64-QAM scheme was obtained for those environments.**

***Keywords*— Rotated constellation, Modulation, Rice fading channel, 64-QAM, Transmission.**

I. INTRODUCTION

In mobile communications, when there is a line of sight (LOS) between transmitter and receiver, the received signal amplitude has a Rice distribution [1]. The relationship between the line of sight and multipath components is given by the Rice factor, K, that is a measure of the transmission quality, with $K = 0$ as the worst case (Rayleigh fading) and $K \to \infty$ representing the absence of fading. A few techniques have been proposed to improve the system performance, including diversity techniques, coded modulation schemes and rotated constellation.

The constellation rotation is a technique that introduces redundancy through a careful choice of a reference angle in a QAM (Quadrature Amplitude Modulation) constellation, combined with the independent interleaving of the symbol components to be transmitted [2], [3], [4]. The technique can improve the performance of mobile communication systems, considering a flat fading channel and the absence of channel estimation errors [5], [6].

This paper presents the analysis of performance for a rotated 64-QAM constellation, the choice of the optimum angle for different environments and the effect of the rotation on the bit error rate (BER).

II. ROTATED CONSTELLATION

A method to reduce the effects of fading is to introduce redundancy with an appropriate choice of the reference angle for a QAM constellation, combined with the independent interleaving of the symbols components to be transmitted, which produces the rotated constellation [2], [3], [4].

The QAM scheme was first proposed by C. R. Cahn in 1960 [7], who extended the phase modulation scheme to phase modulation with multiple amplitudes. In this scheme, the transmitted signal is represented by

$$\begin{aligned} s(t) = &\sum_{n=-\infty}^{+\infty} a_n p(t - nT_S)\cos(\omega_c t) \\ &+ \sum_{n=-\infty}^{+\infty} b_n p(t - nT_S)\sin(\omega_c t), \end{aligned} \tag{1}$$

in which,

$$a_n, b_n = \pm d, \pm 3d, \ldots, \pm(\sqrt{M} - 1)d$$

$$p(t) = \begin{cases} 1, & 0 \le t \le T_S \\ 0, & \text{otherwise,} \end{cases}$$

and ω_c is the carrier frequency.

As shown in Equation 1, the transmitted information in a component is independent of the transmitted information in the other. Furthermore, the signal transmission in independent fading channels can introduce a gain if there redundancy between the two components. The introduction of redundancy in a QAM scheme can be done combining the choice of a reference angle θ of the signal constellation with the interleaving of the independent components [8].

Because of the interleaving process the phase and quadrature components of a transmitted symbol are affected by independent fading. The result of this procedure is an increase in the robustness of the receiver in propagation scenarios with deep fading. After interleaving and rotation the transmitted signal can be written according to

$$\begin{aligned} s(t) = &\sum_{n=-\infty}^{+\infty} x_n p(t - nT_S)\cos(\omega_c t) \\ &+ \sum_{n=-\infty}^{+\infty} y_{n-k} p(t - nT_S)\sin(\omega_c t), \end{aligned} \tag{2}$$

in which, k is an integer that represents the delay (expressed in number the symbols) introduced by interleaving the components I and Q. In addition,

$$x_n = a_n \cos\theta - b_n \sin\theta \tag{3a}$$

and

$$y_n = a_n \sin\theta + b_n \cos\theta \tag{3b}$$

are the new QAM symbols.

The performance gain provided by this technique depends on the fading notches, which are deep but of short duration, and could destroy the information, which is related to the phase and quadrature components of a symbol. This seldom occurs with the rotated constellation, because the symbol components are transmitted during different time intervals and the rotation introduces redundancy between the phase and quadrature components.

The performance gain obtained when using rotated constellations is a function of the rotation angle, and the optimum rotation angle depends on the chosen modulation and channel type [4]. The value of θ does not change the system performance when the transmitted signals are only affected by white Gaussian noise (AWGN channel), since the Euclidean distance between symbols of the constellation does not depend on the angle θ [9].

III. Communication channel with Rice Fading

In terrestrial mobile communications the channel consists of the physical environment between the transmitter and receiver. The presence of obstacles and mobility changes the channel parameters. When different wave components arrive at the mobile receiver with approximately equal amplitudes and uniformly distributed arrival angles, the receiver signal has an envelope with Rayleigh probability distribution.

However, if a component arrives at the mobile receiver, directly or by reflection, with a power that is higher than the others, then the receiver a signal has an envelope with Rice distribution. The component with predominant power is called of direct or specular component [10], [11].

This situation has been commonly observed in the microcell environment [12], and can occur in macro-cells when there is a line of sight transmission (LOS), in mobile communication channels by satellite, among others [13].

The Rice distribution is represented by

$$f_{|u|}(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2 + A_1^2}{\sigma^2}\right) I_0\left(\frac{rA_1}{\sigma^2}\right), r \geq 0. \quad (4)$$

Rewriting $f_{|u|}(r)$ as a function of the root mean square b_R and of the K factor, one obtains

$$f_{|u|}(r) = \frac{2r(K+1)}{b_R}\exp\left(-K - \frac{r^2(K+1)}{b_R}\right) I_0\left(2r\sqrt{\frac{K(K+1)}{b_R}}\right), r \geq 0, \quad (5)$$

Figure 1 represents $f_{|u|}(r)$ for some values of K. For $K = 0$, it represents the Rayleigh curve. As K increases, the curve tends to a Gaussian. Indicating that the frequency at which deep fades occur decreases as the specular component power increases.

A. Computational Modeling

To numerically generate the coefficients of the Rice distribution it is sufficient to add a specular component to the coefficients of the Rayleigh distribution. Thus, the discrete coefficients of the Rice fading can be given by

$$u[n] = L_1(nT_s) + c[n] = A_0 e^{j(w_1 nT_s + \phi_1)} + c[n]. \quad (6)$$

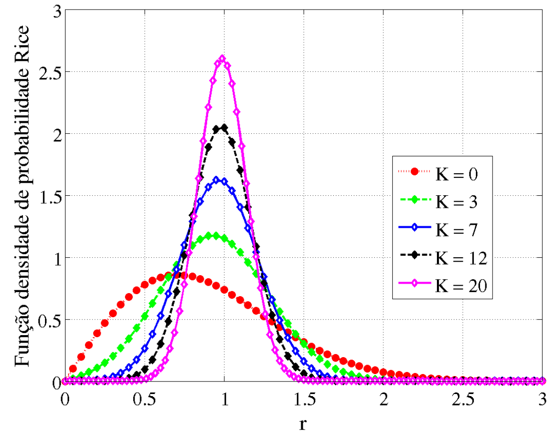

Fig. 1: Rice Probability density function for some values of K, with $b_R = 1$.

From the parameters K and b_R one can obtain A_1 and σ^2 [14]

$$A_1 = \sqrt{\frac{K}{K+1}b_R} \quad \text{and} \quad \sigma^2 = \frac{1}{2(1+K)}b_R. \quad (7)$$

Formula 7 gives the factors of the complex Gaussian process, $\alpha = X + jY$, in which the real and imaginary components have mean μ and standard deviation σ, *i.e.,* $X, Y \sim N(\mu, \sigma^2)$, with mean and standard deviation given by [15], [16]

$$\mu = \sqrt{\frac{K}{2(K+1)}}, \qquad \sigma = \sqrt{\frac{1}{2(K+1)}}. \quad (8)$$

B. Experimental measurements of K

The K factor has been determined experimentally measuring the channel impulse response. In [17], measurements were taken in Ottawa, Canada, at 900 MHz, for a radius of 30 km, with a base station antenna of 33.5 meters high. Four types of environment were considered: open area, suburban, low and medium density urban.

In those environments, the Rice channel model is more appropriate to statistically describe the collected data, indicating the presence of a specular component between the transmitter and the receiver. Table I reproduces some results from [17], they were used to determine the values of K in different environments.

TABLE I: Estimated parameters for the Rice distribution [17].

Environment	A_0	σ	K
Open	0,9615	0,26910	6.38
Suburban	0,9514	0,29960	5.03
Medium-Density Urban	0,9256	0,36167	3.27
Low-Density Urban	0,9022	0,40555	2.48

IV. Evaluation of the Rotated Constellation

This section presents simulations and discussion of results. The simulation considered the rotated constellation and its effect on channels with different Rice parameters, for a 64-QAM scheme.

A. *Evaluation of the optimal angle of rotation*

The evaluation of the optimal angle based in the transmission modulation scheme 64-QAM, which is used in the Brazilian digital television system (ISDB-Tb – Integrated Services Digital Broadcasting Terrestrial built-in) [18], [19].

For the modulation scheme the angles varied from 0 to 45° using the Rice parameters listed in Table I. Figures 2, 3 and 4 indicate the optimal rotation angle for each case.

Note that the angle 16.8° is suitable for transmission with the Rice parameter $K = 6.38$, as can be seen in Figure 4. But for $K = 6.38$, the angles 21° and 32° present the best performance.

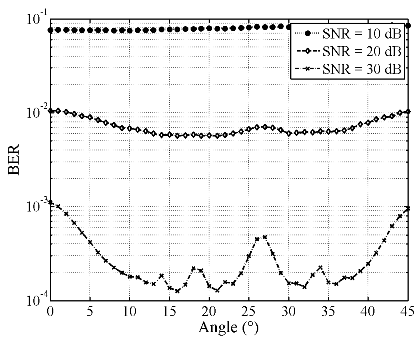

Fig. 2: Plot to obtain the optimum angle for the 64-QAM modulation scheme in a Rice channel with $K = 0$.

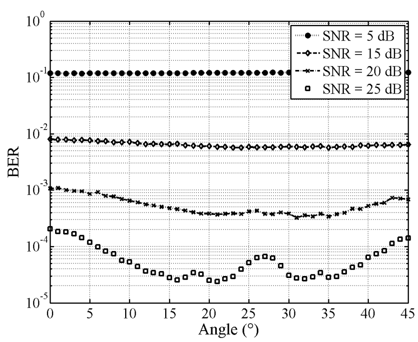

Fig. 3: Plot to obtain the optimum angle for the 64-QAM modulation scheme in a Rice channel with $K = 2.48$.

As the value of K increases, the impact of the phase estimation error becomes less significant to the overall system performance. For example, in Figure 4 the regions near the angles 21° and 32° present a small variation of the Bit Error Ratio (BER).

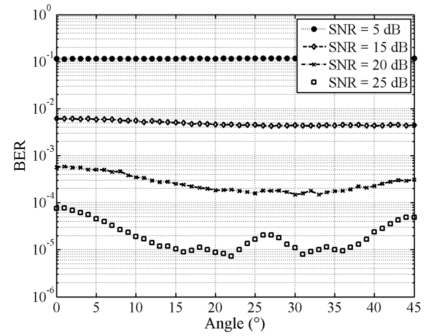

Fig. 4: Plot to obtain the optimum angle for the 64-QAM modulation scheme in a Rice channel with $K = 6.38$.

B. *Evaluation of the Bit Error Rate (BER)*

For comparison purposes, the angle defined for the DVB-T2 (Digital Video Broadcast Terrestrial) [4] was also included in the evaluation, which was performed using BER curves.

The computation of the BER for the 64-QAM modulation scheme, using values for K from Table I, implied that the rotation angle assumes the values 0°, 8,6°, 16°, 21° and 31°. The angle used in the DVB-T2 is 8.6° [4].

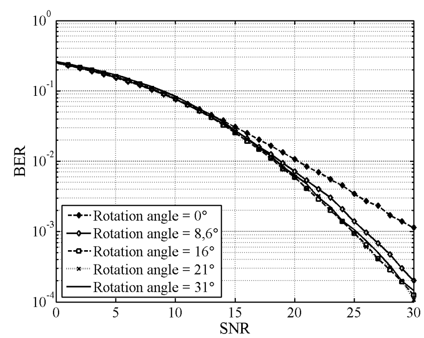

Fig. 5: Plot to obtain the BER for the 64-QAM modulation scheme in a Rice channel with $K = 0$.

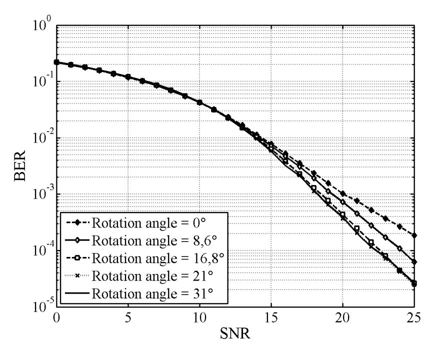

Fig. 6: Plot to obtain the BER for the 64-QAM modulation scheme in a Rice channel with $K = 2.48$.

The case the angle of 8.6° is shown in Figures 5, 6 and 7 angles. For lower SNRs the results are equivalent, and the difference between angles of 31° and 8.6°, in Figure 7, is 2 dB.

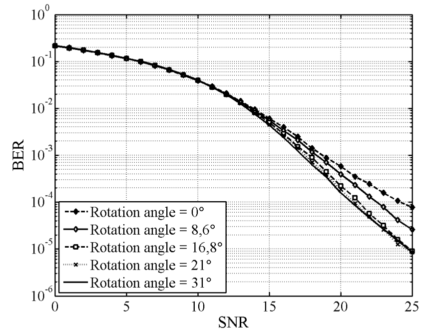

Fig. 7: Plot to obtain the BER for the 64-QAM modulation scheme in a Rice channel with $K = 6.38$.

Comparing the curve for rotation angle of 31°, that presented the best results, with the curve for the original constellation there is a gain of 3 dB. For SNR below 12 dB there is no difference in the curves.

V. Conclusions

This paper presents the technique of the constellation rotation, which improves the performance of the mobile communication systems in communication channels subject to the effects of flat fading. Usually, for mobile communications, the received signal is characterized by the presence of two components: a line of sight (LOS component between the transmitter and the receiver, and the multipath components. This differs from the Rayleigh fading, for which the power of the LOS component is not significantly higher than the multipath components power.

This paper provided an analysis of the values for the optimum angle θ for several values of the K factor, for measurements obtained from five different environments, including the open area environments ($K = 6.38$), suburban area ($K = 5.03$), medium density urban area ($K = 3.27$), low density urban area ($K = 2.48$) and high density urban area ($K = 1$).

The results indicate that is possible to choosing the rotation angle to obtain the best performance for the 64-QAM modulation scheme. The best results were obtained for rotation angles of 16.8°, 21° and 31°. With the rotated constellation, using the best angles, an increase of 8 dB can be obtained over the system without rotation.

Considering the 64-QAM scheme, the best angle for the DVB-T2, with high SNR, is 31°, as compared to the usual DVB-T2 angle (8.6°), and the obtained gain 2 dB.

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