On the Fading Parameter Characterization of the κ-μ Extreme Distribution

Camila Lopes Selvati and Ugo Silva Dias

***Abstract*— Several traditional models exist to describe the multipath phenomenon in wireless communications, such as Rice, Nakagami-*m*, Hoyt, and Rayleigh. There are other new ones that propose a generalization in order to better fit fading environments, such as α-μ, κ-μ and η-μ. In this work, the κ-μ extreme distribution, a special case of the κ-μ fading channel, is investigated. The statistical characterization of the κ-μ extreme fading parameter, m, is presented, considering both indoor and outdoor field trial measurements. More specifically, the empirical probability density function and the autocorrelation function of the fading parameter of the κ-μ extreme distribution are obtained. Moreover, from the empirical data, the range of possible practical values is also obtained and discussed. Finally, the instantaneous magnitude variation of m considering the mobile receiver displacement is estimated. The results presented here provide important information about the actual usefulness of the κ-μ extreme fading model in mobile communication systems.**

***Keywords*— κ-μ extreme distribution, channel characterization, fading parameter, field measurements.**

I. Introduction

THE performance of wireless communications systems is harshly depreciated by the multipath fading phenomenon caused by different types of obstacles between transmitter and receiver. Various important statistical models characterize this phenomenon, notably Rayleigh [1], Rice [2], [3], Hoyt (Nakagami-*q*) [4], [5], Nakagami-*m* [5], and Weibull [6]. In [7], the κ-μ extreme fading distribution was proposed as a particular case (its parameters assume extreme values [8]) of the κ-μ distribution, which describes small-scale variations of the fading signal under a light-of sight (LOS) condition. The κ-μ distribution regards important other models as special cases such as Rice (Nakagami-*n*) and Nakagami-*m* [7]. Hence, One-Sided Gaussian and Rayleigh also constitute special cases of it. Its flexibility renders it suitable to better fit field measurement data in a variety of scenarios, both for low- [7], [9] and high-order statistics [10]. The same happens for the κ-μ extreme distribution which is suitable to better fit trial measurements in several environments when the dominant component is very high and the multipath clustering is extremely low [8]. Experimental data supporting the usefulness of the Nakagami-*m* and Weibull fading models have been vastly reported in the literature (e.g., [11]–[14]). Recently [15]–[17], some works presenting the performance analysis and first-order statistical modeling for the Nakagami-*m* and Rice fading parameters have also been presented. In [8] the κ-μ extreme distribution had its statistics obtained, nonetheless, works depicting the statistical characterization of the fading parameter of the distribution, using practical data, are not known by the authors.

C. L. Selvati and U. S. Dias are with the Microwave and Wireless Systems Laboratory (MWSL), Department of Electrical Engineering, University of Brasilia, 70910-900, Brasilia, DF, Brazil (e-mail: camilaselvati@ieee.org, udias@unb.br).

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The main purpose of this paper is to obtain the empirical probability density function (PDF) and its best theoretical fit, and the autocorrelation function of m, which is the fading parameter of the κ-μ extreme distribution, based on field measurements. Furthermore, the evaluation of the range of possible practical values assumed by m and its instantaneous magnitude variation are estimated from the empirical data considering the displacement of the mobile receiver in different indoor and outdoor environments. The knowledge of the parameter's PDF, autocorrelation, and magnitude range is significant because the value of m is a multipath clustering indicator of the radio channel [7] and can be used in the evaluation and design of different wireless communications techniques, such as diversity combining techniques, adaptive modulation schemes, modeling and analysis of interferences, outages probabilities, design and simulation of adaptive antennas systems, among others.

The remainder of this work is structured as follows. In Section II, κ-μ extreme distribution is revisited. In Section III, indoor and outdoor field trial measurements are conducted in order to investigate first- and high-order statistics of the m fading parameter. Specially, the empirical PDF and autocorrelation of the m fading parameter are presented and discussed based on the measurement campaigns. The instantaneous variation of the magnitude of m with distance is also obtained and analyzed, and the practical value range of m is estimated. Finally, in Section IV, some conclusion remarks are presented.

II. The κ-μ Extreme Distribution Revisited

The κ-μ extreme distribution is one of the various cases of the κ-μ distribution, which is a general fading distribution that can be used to better represent small-scale variations of the fading signal under a LOS condition [7]. The κ-μ distribution includes as special cases important other distributions, such as Rice (Nakagami-*n*) and Nakagami-*m* [7]. (Therefore, One-Sided Gaussian and Rayleigh are also special cases of it). As its name connotes, it is defined in terms of two physical parameters, namely κ and μ. The parameter $\kappa > 0$ refers to the ratio between the total power of the dominant components and the total power of the scattered waves, whereas the parameter $\mu > 0$ is related to the multipath clustering. The κ-μ

extreme distribution is obtained by manipulating these physical parameters for a certain amount of fading [8]. Specifically, the Nakagami-*m* parameter *m* is obtained when $\kappa \to 0$. In such case, $\mu = m$.

For a κ-μ fading signal with envelope R and a normalized envelope $\mathrm{P} = R/\hat{r}$, with $\hat{r} = \sqrt{\mathrm{E}(R^2)}$, the κ-μ envelope PDF, $f_{\mathrm{P}}(\rho)$, is written as [7]

$$f_{\mathrm{P}}(\rho) = \frac{2\mu(1+\kappa)^{\frac{\mu+1}{2}}}{\kappa^{\frac{\mu-1}{2}}\exp(\kappa\mu)}\rho^{\mu}\exp\left[-\mu(1+\kappa)\rho^2\right] \times I_{\mu-1}\left[2\mu\sqrt{\kappa(1+\kappa)}\rho\right], \quad (1)$$

for which $\mu > 0$ is given by [7]

$$\mu = \frac{\mathrm{E}^2(R^2)}{\mathrm{Var}(R^2)}\frac{(1+2\kappa)}{(1+\kappa)^2}, \quad (2)$$

$\mathrm{I}_{\nu}(\cdot)$ is the modified Bessel function of the first kind and order ν [18, Equation 9.6.20], $\mathrm{Var}(\cdot)$ and $\mathrm{E}(\cdot)$ denote the variance and the expectation operators, respectively.

Moreover, it is known from [7] that κ and μ can be expressed in terms of the normalized variance of the power of the fading signal, usually defined as *m*. In other words, [8]

$$m = \frac{\mu(1+\kappa)^2}{1+2\kappa}. \quad (3)$$

Given that $\mu > 0$ and $\kappa > 0$, and that a relationship among κ, μ and *m* is found through Equation 3, for a fixed m, as $\mu \to 0$ then $\kappa \to \infty$. In such case, it can be shown that the κ-μ extreme PDF is given by [8]

$$f_{\mathsf{P}}(\rho) = \frac{4mI_1(4m\rho)}{\exp[2m(1+\rho^2)]} + \left[1 - \frac{\sqrt{2m\pi}}{\exp(m)}I_{0.5}(m)\right]\delta(\rho). \quad (4)$$

III. Field Trials and Statistical Analysis

A series of field trials was conducted at the University of Campinas (Unicamp), Brazil, in order to (i) estimate the value range of the m parameter, (ii) characterize the amplitude variation with distance of the m fading parameter, (iii) obtain the empirical PDF and the theoretical fit of the m parameter, and (iv) obtain the empirical autocorrelation function of such fading parameter. To this end, the transmitter was placed on the rooftop of one of the buildings as shown in Figure 1, and the receiver traveled through campus in different environments such as the ones shown in Figures 2 to 4 as well as within the buildings. The mobile reception equipment was especially assembled for this purpose. Basically, the setup consisted of a vertically polarized omnidirectional receiving antenna, a low noise amplifier, a spectrum analyzer, data acquisition apparatus, a notebook computer, and a distance transducer for carrying out the signal sampling. The transmission consisted of a continuous wave (CW) tone at 1.8 and 5.5 GHz. The spectrum analyzer was set to zero span and centered at the desired frequency, and its video output used as the input of the data acquisition equipment with a sampling interval of $\lambda/14$ [19]–[21] for 1.8 GHz and $\lambda/180$ [22], [23] for

Fig. 1. Building 'E' – Faculty of Electrical and Computing Engineering. Transmitter on the rooftop.

Fig. 2. Gymnasium. Indoor measurements.

Fig. 3. Hospital das Clínicas. Outdoor measurements.

5.5 GHz. The local mean was estimated by the moving average method, with the average being conveniently taken over samples symmetrically adjacent to every point. From the collected data, the long term fading was filtered out, then the m fading parameter could be estimated.

A. The Empirical Autocorrelation

The normalized empirical autocorrelation was computed according to

Fig. 4. Cora Coralina Street. Outdoor measurements.

$$\widehat{A}_R(\Delta) = \frac{\sum_{i=1}^{N-\Delta} r_i r_{i+\Delta}}{\sum_{i=1}^{N-\Delta} r_i^2}, \tag{5}$$

in which r_i is the i-th sample of the amplitude sequence, N is the total number of samples, Δ is the m discrete relative distance difference, and $\widehat{A}_R(.)$ denotes an empirical estimate of $A_R(.)$.

B. Numerical Results and Discussion

Figures 5 to 7 show sample plots of the magnitude variation of the m fading parameter with distance for the same indoor and outdoor environments presented in Figures 2 to 4. It must be said that peak values (highest values assumed by m) occur when the dominant component is weak and there is a prevalence of multipath clusters. Conversely, the lowest values for m occur when the dominant component appears in the propagation path [Eq. (3)].

Figures 8 to 10 present sample curves of the empirical m PDF for the same environments of Figures 5 to 7. Also, it is shown the PDF fitting with the theoretical κ-μ extreme PDF. The values that m assume for which the κ-μ extreme theoretical PDF fits the empirical PDF of m vary from 0.96 to 1.27. At 1.8 GHz, the average estimated median and mean values of m considering all measurement campaigns are respectively 1.351 and 1.345, with standard deviations of 0.359. Whereas at 5.5 GHz, the average estimated median and mean values of m considering all measurement campaigns are respectively 1.217 and 1.319, with standard deviations of 0.491. The range of possible practical values of m, as found here, vary from 0.586 to 2.362 for 1.8 GHz and 0.605 to 2.588 for 5.5 GHz. The knowledge of possible practical levels of m and its expected value is very important to better estimate the fading margin in wireless systems.

Finally, Figures 11 to 13 depict sample plots of the empirical autocorrelation function of the m fading parameter for different indoor and outdoor environments and frequencies. Interestingly, observe how the autocorrelation curves tend to keep track of the changes of the concavity.

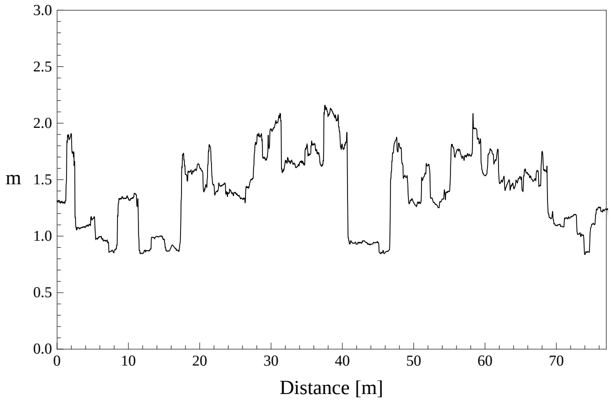

Fig. 5. m magnitude with distance. Indoor measurements at 1.8 GHz

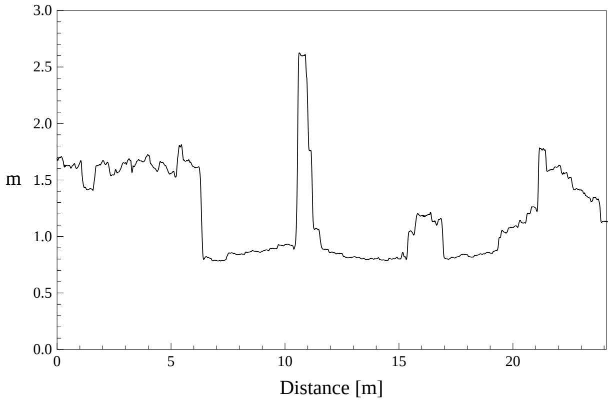

Fig. 6. m magnitude with distance. Outdoor measurements at 1.8 GHz

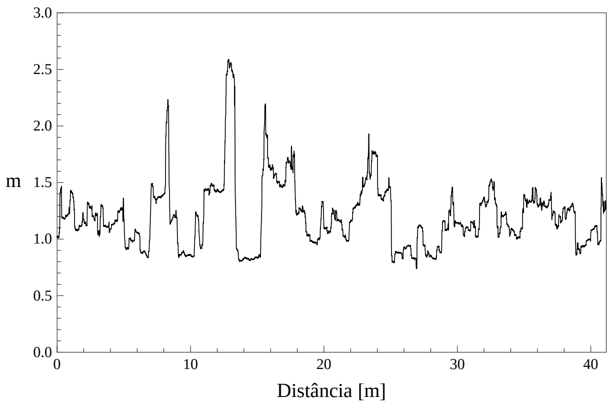

Fig. 7. m magnitude with distance. Outdoor measurements at 5.5 GHz

IV. Conclusions

This work presented the statistical characterization of the fading parameter of the κ-μ extreme distribution. More specifically, the results of field trials aimed at investigating first- and second-order statistics were obtained, such as the empirical probability density function and the autocorrelation function of the m fading parameter. In addition, the range of possible practical values for the fading parameter was estimated from the empirical data, and the instantaneous variation of its magnitude was evaluated considering the displacement of the mobile receiver in indoor and outdoor environments. The results obtained here provide important information about the practical usefulness of the κ-μ extreme fading model in mobile wireless communication systems.

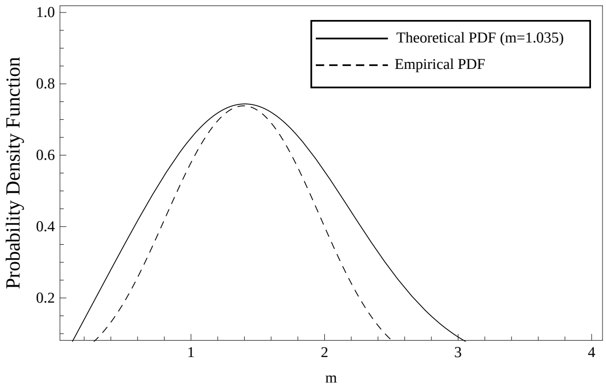

Fig. 8. Theoretical and empirical PDFs of the m fading parameter. Indoor measurements at 1.8 GHz

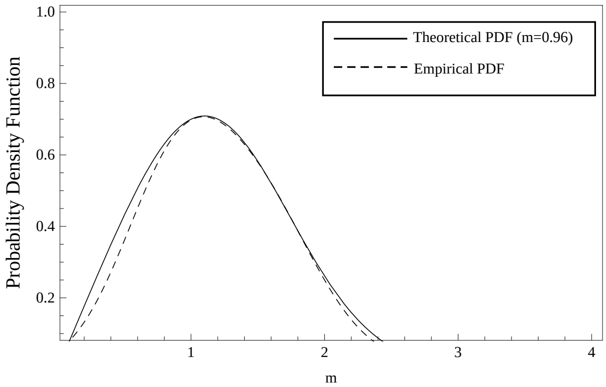

Fig. 9. Theoretical and empirical PDFs of the m fading parameter. Outdoor measurements at 1.8 GHz

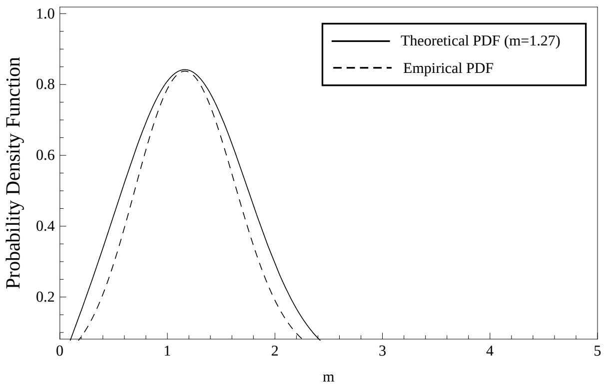

Fig. 10. Theoretical and empirical PDFs of the m fading parameter. Outdoor measurements at 5.5 GHz

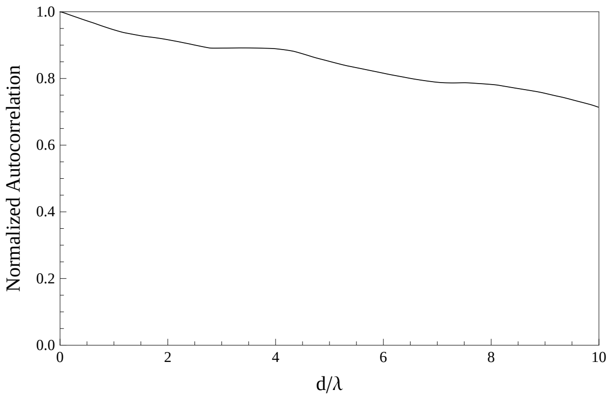

Fig. 11. Empirical autocorrelation of the m fading parameter. Indoor measurements at 1.8 GHz

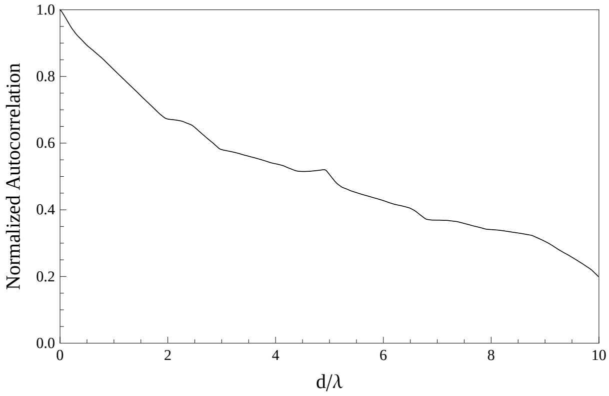

Fig. 12. Empirical autocorrelation of the m fading parameter. Outdoor measurements at 1.8 GHz

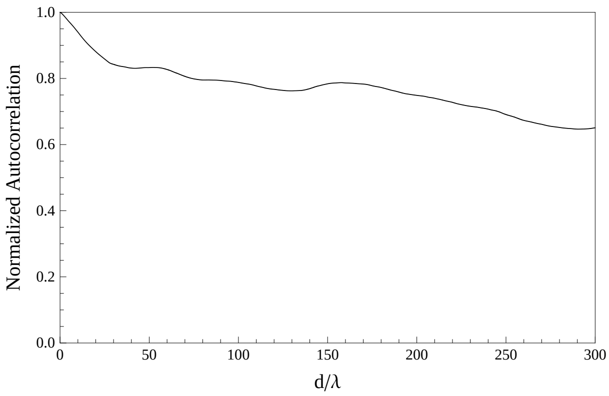

Fig. 13. Empirical autocorrelation of the m fading parameter. Outdoor measurements at 5.5 GHz

REFERÊNCIAS

[1] R. H. Clarke, "A statistical theory of mobile-radio reception," *Bell Syst. Tech. J.*, vol. 47, pp. 957–1000, Jul.–Aug. 1968.

[2] S. O. Rice, "Mathematical analysis of random noise," *Bell Syst. Tech. J.*, vol. 23, no. 3, pp. 282–332, Jul. 1944.

[3] ——, "Mathematical analysis of random noise," *Bell Syst. Tech. J.*, vol. 24, no. 1, pp. 46–156, Jan. 1945.

[4] R. S. Hoyt, "Probability functions for the modulus and angle of the normal complex variate," *Bell Syst. Tech. J.*, vol. 26, pp. 318–359, Apr. 1947.

[5] M. Nakagami, *The m-Distribution – a general formula of intensity of rapid fading*. Statistical Methods in Radio Wave Propagation, Permagon Press, 1960.

[6] M. D. Yacoub, D. B. da Costa, U. S. Dias, and G. Fraidenraich, "Joint Statistics for Two Correlated Weibull Variates." *IEEE Ant. and Wireless Prop. Lett.*, vol. 4, pp. 129–132, May 2005.

[7] M. D. Yacoub, "The κ-μ and the η-μ distribution," *IEEE Antennas Propag. Mag.*, vol. 49, no. 1, pp. 68–81, Feb. 2007.

[8] G. S. Rabelo and M. D. Yacoub, "The κ-μ Extreme Distribution," *IEEE Trans. on Comm.*, vol. 59, no. 10, pp. 2776–2785, Oct. 2011.

[9] U. S. Dias and M. D. Yacoub, "The κ-μ Joint Phase-Envelope Distribution," *IEEE Trans. Commun.*, vol. 58, no. 1, Jan. 2009.

[10] S. L. Cotton and W. G. Scanlon, "High-order statistics for κ-μ distribution," *IEE Electron. Lett.*, vol. 43, no. 22, Oct. 2007.

[11] U. S. Dias, M. D. Yacoub, J. C. S. Santos Filho, G. Fraidenraich, and D. B. da Costa, "On the Nakagami-*m* Autocorrelation and Power

Spectrum Functions: Field Trials and Validation," in *IEEE International Telecommunications Symposium, ITS'06*, Fortaleza-CE, Brazil, Sep. 2006, pp. 253–256.

[12] H. Hashemi, "The indoor radio propagation channel," *Proc. IEEE*, vol. 81, pp. 943–968, Jul. 1993.

[13] G. Tzeremes and C. G. Christodoulou, "Use of Weibull distribution for describing outdoor multipath fading," in *Proc. IEEE Antennas and Propagation Soc. Int. Symp.*, vol. 1, Jun. 2002, pp. 232–235.

[14] F. Babich and G. Lombardi, "Statistical analysis and characterization of the indoor propagation channel," *IEEE Trans. Commun.*, vol. 48, pp. 455–464, Mar. 2000.

[15] L. Rubio, J. Reig, and N. Cardona, "Evaluation of nakagami fading behaviour based on measurements in urban scenarios," *AEU - International Journal of Electronics and Communications*, vol. 61, no. 2, pp. 135 – 138, 2007.

[16] L. Greenstein, S. Ghassemzadeh, V. Erceg, and D. Michelson, "Ricean K-Factors in Narrow-Band Fixed Wireless Channels: Theory, Experiments, and Statistical Models," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 8, pp. 4000 –4012, oct. 2009.

[17] C. Oestges, B. Villacieros, and D. Vanhoenacker-Janvier, "Radio Channel Characterization for Moderate Antenna Heights in Forest Areas," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 8, pp. 4031 –4035, oct. 2009.

[18] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. New York: Dover, 1972.

[19] B. Davis and R. Bogner, "Propagation at 500 MHz for mobile radio," *IEE Proc.*, vol. 132, pp. 307–320, Aug. 1985.

[20] K. T. J. Horikoshi and T. Morinaga, "1.2 GHz Band Wave Propagation Measurements in Concrete Building for Indoor Radio Communications," *IEEE Trans. Veh. Technol.*, vol. 35, no. 4, pp. 146–152, Aug. 1986.

[21] A. F. Toledo, "Narrowband characterization of radio transmissions into and within buildings at 900, 1800, and 2300 MHz," May 1992.

[22] B. Davis, and R. Bogner, "Propagation at 500 MHz for mobile radio," *IEEE Proc.*, vol. 132, Aug. 1985.

[23] K. T. .J. Horikoshi and T. Morinaga, "1.2 GHz Band Wave Propagation Measurements in Concrete Buildings for Indoor Radio Communication," *IEEE Trans. Veh. Tech.*, vol. 35, Aug. 1986.