On the α-μ/Gamma Composite Distribution: Field Trials and Validation

Sérgio Pinho Piretti, Pedro Carlos Batista Bretas Jr. and Ugo Silva Dias

***Abstract*— This work proposes and conduct a practical investigation on the a probability density function of the α-μ/Gamma composite distribution, obtained in closed-form expression, for the multipath shadowed fading channel in wireless environments. The statistical characterization includes both theoretical models and practical investigations through field measurements. More specifically, the analyses was realized for the mobile radio channel operating in the frequencies 780 MHz, 1800MHz and 2500 MHz. A transmission and reception system was specially designed for field measurements and a software for processing the captured data was built. The practical data are confronted with the theoretical data obtained from the α-μ/Gamma distributions and very good fits are found.**

***Keywords*— Composite Fading, Channel Characterization, Field Measurements, α-μ/Gamma.**

I. INTRODUCTION

The performance of wireless systems are severely penalized by the stochastic nature of the mobile radio channel. In the path between transmitter and receiver, besides the propagation loss, the mobile-radio signal can be blocked by physical obstructions - shadowing - and suffer multiple reflections, scattering and diffraction - multipath fading.

The mean of the signal envelope at the mobile station (MS), the envelope averaged over a distance of a few wavelengths, is a random variable due to shadowing (slow fading). The probability density function (PDF) of the envelope mean is widely accepted to be lognormal [4]. While the multipath fading (fast fading) have been described by a numerous distributions like Rayleigh, Rice, Nakagami-m, Weibull and others. This multipath fading models generally assume constant average signal power. However, in some situations like congested downtown areas with slow moving pedestrians and vehicles, received average power also become random. As result, fading environment consist of multipath fading superimposed on shadowing, called as the shadowed fading channel [1] [2].

Various composite distributions have been suggested in the literature to model this compound phenomenon. Perhaps, the most known is the Rayleigh-Lognormal model also called as Suzuki Model, witch consists on the compound between the short fading Rayleigh with shadowing Lognormal. The main inconvenience of this model is that your composite PDF is not a closed expression, witch makes performance evaluations, such as average error probability, outage probability and channel capacity, be mathematically complicated. Abdi in [4] proposes the use of the Gamma distribution as an alternative to the lognormal distribution in describing the shadowing phenomenon. And he also apply this alternative to make the K-distribution as an option for the Rayleigh-Lognormal composite fading. The K-distribution has limits flexibility such that it is unable to fit different levels of composite fading-shadowing scenarios.

The α-μ, presented in [3], have been proposed in order to provide a more realistic analysis of the propagated signal. This fast fading model has two degrees of freedom, if compared to Rayleigh, achieving greater flexibility. The α-μ/Gamma distribution was first presented in [7] with PDF described in terms of MeijerG special function. As said in [1, Seq. 2.2.1.5], most modulation/detection schemes can not obtain closed-forms expressions for performance analyses, such as average error probability, because of the difficulty on evaluating integrals whose integrand involves Meijer's G-function. So this paper rewrites the α-μ/Gamma distribution in a closed-form fashion expression not in terms of MeijerG special function and presents a practical investigation for it in the frequencies 780 MHz, 1800MHZ and 2500 MHz.

The remainder of this article is organized as follows. Section II revisits the α-μ and the Gamma distributions. In Section III, we present the model formulation used to obtain the composite distribution, the α-μ/Gamma PDF and some numerical results. Section IV describes how the field trials were conducted for the practical investigation of the composite model. Section V discuss the results of the practical investigation and in Section VI some final conclusions can be found.

II. THE α-μ AND GAMMA DISTRIBUTIONS

A. The α-μ Fading Model

The α-μ distribution considers a signal compose of clusters of multipath waves propagating in a non homogeneous environment. Each *clusters* is characterized for scattered waves with identical powers, random phases and similar delay times. For different *clusters*, the delay-time spreads are relatively large. The resulting envelope is obtained as a nonlinear function of the modulus of the sum of the multipath components. Such a nonlinearity is manifested in terms of a power parameter, $\alpha > 0$ and the numbers of multipath clusters is related to the parameter $\mu > 0$. This distribution includes a especial case of Nakagami-m (α = 2 and μ = m) and Weilbull (α = 1)[3].

For a fading signal with envelope R, an arbitrary parameter $\alpha > 0$, and a α-root mean value $\hat{r} = \sqrt[\alpha]{E[R^\alpha]}$, the α-μ envelope PDF, $f_R(r)$ of R, is written as

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$$f_R(r) = \frac{\alpha\mu^{\mu} r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu}\Gamma[\mu]} \exp\left[-\mu\frac{r^{\alpha}}{\hat{r}^{\alpha}}\right], \quad (1)$$

in which $\Gamma[\cdot]$ is the Gamma function, $E[\cdot]$ is the expectation operator, $V[\cdot]$ is the variance operator and $\mu > 0$ is the inverse of the normalized variance of R^{α}, i.e.,

$$\mu = \frac{E^2[r^{\alpha}]}{V[r^{\alpha}]}. \quad (2)$$

For the normalized envelope $\rho = R/\hat{r}$, the PDF $f_P(\rho)$ of P is obtained as

$$f_P(\rho) = \frac{\alpha\mu^{\mu}\rho^{\alpha\mu-1}}{\Gamma[\mu]} \exp[-\mu\rho^{\alpha}], \quad (3)$$

in which $\mu > 0$ is given by

$$\mu = \frac{1}{V[P^{\alpha}]}. \quad (4)$$

The moment of the α-μ distribution also described in [3] is written as $E[R^k] = \frac{\hat{r}^k\Gamma[\mu+k/\alpha]}{\mu^{k/\alpha}\Gamma[\mu]}$ and together with (2) is used do define an equality useful to estimate the parameters of this distribution. The moment-based α-μ-estimator is described as

$$\frac{E^2[R^{\beta}]}{E[R^{2\beta}] - E^2[R^{\beta}]} = \frac{\Gamma^2[\mu+\beta/\alpha]}{\Gamma[\mu]\Gamma[\mu+2\beta/\alpha] - \Gamma^2[\mu+\beta/\alpha]}, \quad (5)$$

in which β is an arbitrary parameter. In order to estimate the physical parameters α and μ, a system of two equations can be configured, once two distinct values of β have been chosen arbitrarily. For a particular case in which β = 1 and β = 2, (4) yields an estimator in terms of the first and second moments [8].

B. The Gamma Fading Model

As show in [4] and [6], the lognormal distribution is widely accepted as the optimum statistical model for characterizing the shadowing phenomenon, however it is not easy-to-use when it is involved in closed-form composite PDFs for fading channels. And in this cases the Gamma distribution can be used as an approximation of the lognormal distribution. The envelope PDF of Gamma is written in [6] as

$$f_{\Omega}(y) = \frac{1}{\Gamma[m_s]}\left(\frac{m_s}{\Omega_s}\right)^{m_s} y^{m_s-1}\exp\left[-y\frac{m_s}{\Omega_s}\right], \quad (6)$$

the parameter m_s inversely reflects the shadowing severity and the parameter Ω_s is the gamma shadow area mean power.

To estimate values for m_s and Ω_s, the following expressions can be used

$$m_s = \frac{E[y]^2}{V[y]}, \quad (7)$$

$$\Omega_s = E[y]. \quad (8)$$

For a normalized envelope $\rho_s = y/\Omega_s$, the probability density function ρ_s is obtained as

$$f_P(\rho_s) = \frac{{m_s}^{m_s}}{\Gamma[m_s]}{\rho_s}^{m_s-1}\exp[-\rho_s m_s]. \quad (9)$$

III. The α-μ/Gamma Fading Distribution

A. Model Formulation

A composed probability density function can be derived by superimposing two or more statistical distributions. This method is expressed as follows

$$f_X(x) = \int_0^{\infty} f_{X|Y}(x|y) f_Y(y) dy. \quad (10)$$

For the composite multipath/shadowing fading model of this article, the superimposing is done with the fast and the shadowing fading distributions in which the first is $f_{X|Y}(x|y)$ and the second is $f_Y(y)$. Setting $\hat{r}^{\alpha} = y$ in (1), we get the following integral

$$f_R(r_c) = \int_0^{\infty} \frac{\alpha\mu^{\mu} r_c^{\alpha\mu-1}}{y^{\mu}\Gamma[\mu]}\exp\left[-\mu\frac{r_c^{\alpha}}{y}\right] \times \frac{1}{\Gamma[m_s]}\left(\frac{m_s}{\Omega_s}\right)^{m_s} y^{m_s-1}\exp\left[-y\frac{m_s}{\Omega_s}\right] dy. \quad (11)$$

The random variable r_c represents the composite envelope. The parameters α, μ, m_s and Ω_s are the same used in (1) and (6) and are estimated the same way.

The process is only possible because the y represents in both distributions the mean power of the fast fading, in other words, the same physical entity. Solving the integral we have the following expression, in which $K_n[\cdot]$ is the modified Bessel function of the second kind and order n,

$$f_R(r_c) = \frac{2\alpha}{\Gamma[m_s]\Gamma[\mu]}\left(\frac{m_s\mu}{\Omega_s}\right)^{\frac{m_s+\mu}{2}} r_c^{-1+\frac{1}{2}\alpha(m_s+\mu)} \times K_{m_s-\mu}\left[2r_c^{\frac{\alpha}{2}}\sqrt{\frac{m_s\mu}{\Omega_s}}\right]. \quad (12)$$

For the normalized envelope $\rho_c = r_c/\sqrt[\alpha]{\Omega_s}$, the α-μ/Gamma PDF $f_P(\rho_c)$ is

$$f_P(\rho_c) = \frac{2\alpha}{\Gamma[m_s]\Gamma[\mu]}(m_s\mu)^{\frac{m_s+\mu}{2}}\rho_c^{-1+\frac{1}{2}\alpha(m_s+\mu)} \times K_{m_s-\mu}[2\rho_c^{\frac{\alpha}{2}}\sqrt{m_s\mu}]. \quad (13)$$

By setting α = 2 and μ = m, we obtain the Nakagami-m/Gamma described in [6, Eq. 9], setting α = 2 and μ = 1, the distribution reduces to Rayleigh/Gamma [5] and the Weilbull/Gamma distribution also can be obtained by setting the parameter μ = 1.

B. Numerical Results

A family of curves for the PDF $f_P(\rho_c)$ *versus* ρ_c, with $\mu = 4/7$, $m_s = 3$ and α as a variable parameter is plotted in Figure 1.

Figure 2 also shows the PDF $f_P(\rho_c)$, however the multipath parameter μ is varied while $\alpha = 7/4$ and $m_s = 3$. A third family of curves is plotted in Figure 3. For this case, the shadowing parameter m_s receives multiples values whilst $\alpha = 2.5$ e $\mu = 1.5$. For comparison, the corresponding no shadowing, $m_s \to \infty$, are also shown.

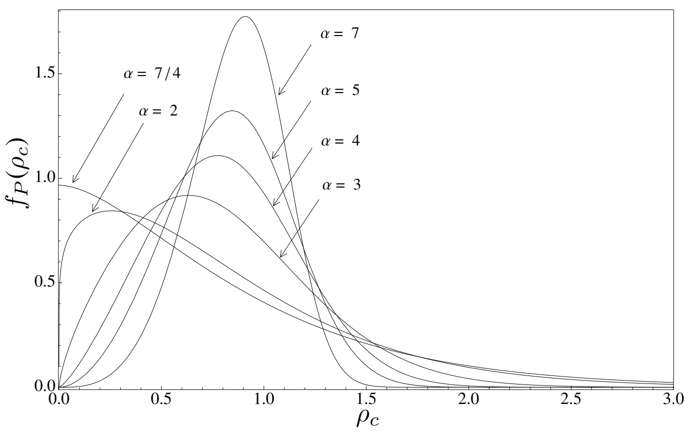

Fig. 1. Various shapes for the α-μ/Gamma density function for $\mu = 4/7$ and $m_s = 3$.

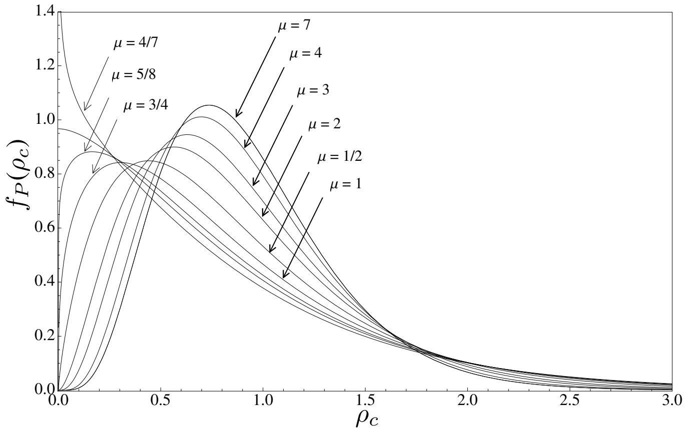

Fig. 2. Various shapes for the α-μ/Gamma density function for $\alpha = 7/4$ and $m_s = 3$.

IV. Field Trials

Series of outdoors field trials were conducted at the University of Brasília (UnB) and at the University of Campinas (Unicamp), Brazil, in order to obtain the empirical PDFs of the composite multipath/shadowing fading and then use them for a practical investigation of the α-μ/Gamma distribution. To this end, the transmitter was always placed on the rooftop of a building and the receiver travelled through the universities. The mobile reception equipment was especially assembled for this purpose. Basically, the setup consisted of a vertically polarized omnidirectional receiving antenna, a low noise amplifier, a spectrum analyzer, a data acquisition equipment and a notebook computer. For the trials conducted at UnB, the receiver setup was installed in a car moving with constant velocity while the trials at Unicamp used a pushcart equipped with a distance transducer for carrying out the signal sampling. The transmission consisted of a continuous wave tone at 2500 MHz and 780MHz at UnB and at 1800MHz at Unicamp. The spectrum analyzer was set to zero span and centered at the desired frequency, and its video output used as the input of the data acquisition equipment with a sampling interval of 300 samples per second. The local mean was estimated by the moving average method, with windows length of 45λ [9]. From the collected data, the short term, the long term and the path loss were isolated, then a composite envelope was made by adding the short and the long terms.

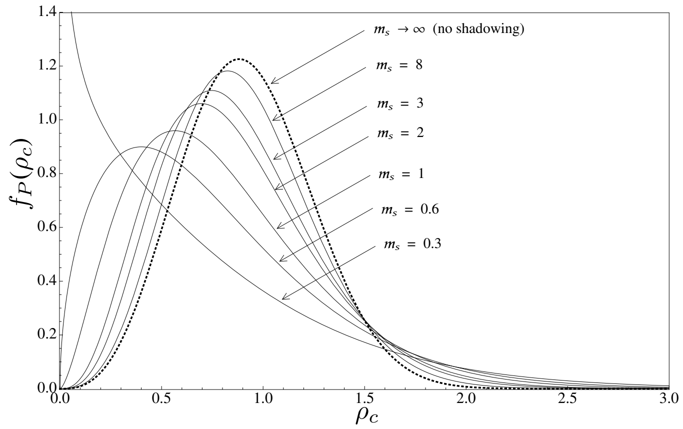

Fig. 3. Various shapes for the α-μ/Gamma density function for $\alpha = 2.5$ and $\mu = 1.5$.

V. Practical Investigation and Discussions

In this section we present two comparative results for each frequency used in the field trails. In the figures 4, 5 and 6, the α-μ/Gamma PDF curves were plotted together with the experimental PDF and the errors between the theoretical and the experimental PDF were calculated in order to do a numerical evaluation among the trials. The error calculation was made according to the equation

$$\varepsilon_{PDF} = E[f_{emp} - f_{dist}], \tag{14}$$

in which f_{dist} represents the α-μ/Gamma distribution PDF and the f_{emp} is the empirical PDF.

It is possible to observe a very good fit between the theoretical and the empirical PDFs for each of the frequencies investigated.

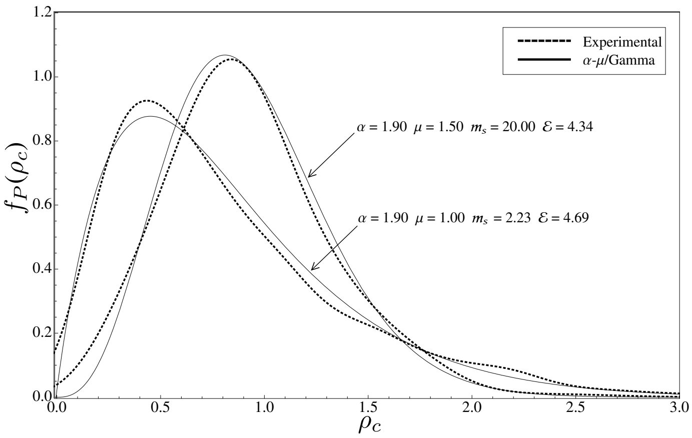

Fig. 4. Comparison between the theoretical and empirical PDFs for composite fading at 780 MHz.

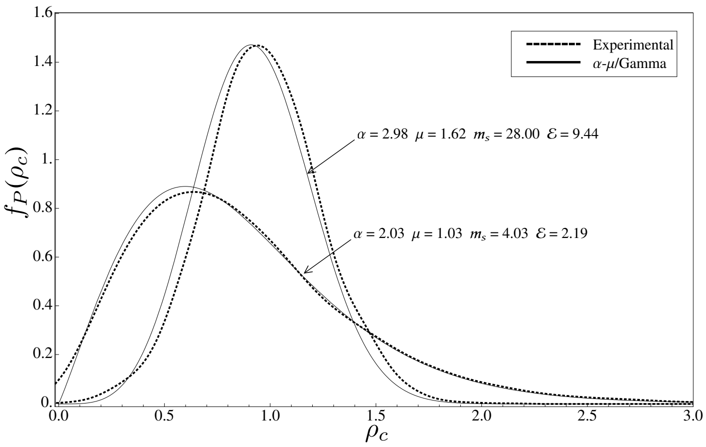

Fig. 5. Comparison between the theoretical and empirical PDFs for composite fading at 1800 MHz.

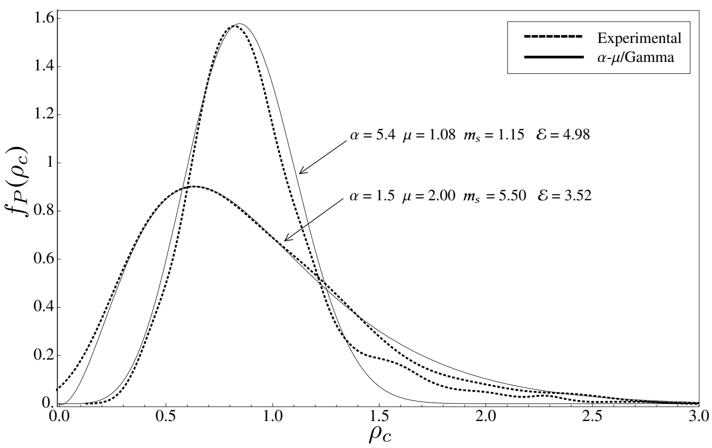

Fig. 6. Comparison between the theoretical and empirical PDFs for composite fading at 2500 MHz.

VI. Conclusions

In order to model the wireless channel subject to both long term and short term fading, this work obtained a closed-form fashion expression for the α-μ/Gamma distribution which has as special cases the Rayleigh/Gamma, Nakagami-m/Gamma and Weibull/Gamma. A practical investigation was conduct at three different frequencies, the data collected were confronted with the theoretical data and very good fits were found between your PDFs. The probability density function of this distribution can be considered as a useful mathematical tool in applications related to performance evaluation of wireless communications under composite channels.

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