Co-Channel Interference Effects On Spatial Multiplexing

Juan Carlos Minango and Celso de Almeida *

***Abstract*—When spatial multiplexing (SM) technique is applied to a cellular system, the performance is affected by the interference from neighboring co-channel cells. This paper investigates and compares the performance between SM and non-SM systems in a noise-limited and interference-limited environment, where parameters related to the transmit power and spectral efficiency are taken into consideration, in order to make a fair comparison between the both systems. Monte Carlo simulations were performed for obtaining the results in terms of the bit error rate (BER) as a function of per-bit signal-to-noise ratio (E_b/N_o) and signal-to-interference ratio (SIR).**

***Index Terms*—Spatial Multiplexing, BER, Co-Channel Interference.**

I. Introduction

New generation of mobile communication system demands more and more broadband services. However, the available bandwidth is limited.

Spatial multiplexing (SM) is a powerful technique used to increase the transmission rate without bandwidth expansion [1], [2]. This technique divides the incoming data into multiple parallel substreams and transmits each on a different spatial dimension (e.g., a different antenna).

In a cellular network, multiple antennas for SM are in general collocated at base stations (BS) [3], because high data rates are particularly interesting for the downlink.

Another important factor in a cellular network is that the same channel is reused in spatially separated cells to efficiently utilize the limited frequency bandwidth. Therefore, the receiver suffers from co-channel interference (CCI) from neighboring cells [4].

The majority of studies of SM has focused on the point-to-point model [5], [6], which ignores CCI. In this paper, we present a simulation study of SM in terms of the bit error rate (BER) for a cellular network with CCI in the downlink.

Section II shows the system model description, section III describes the optimum decoder, section IV presents the performance analysis, where BER expressions for M-ASK and M-QAM modulations in the presence of one interferer are obtained, section V shows the simulation results in terms of BER for noise-limited and interference-limited environment, and finally, section VI presents the conclusions.

II. System Model

We consider a spatial multiplexing communication system in a noise-limited environment (see Fig. 1), which transmits symbols. If N_t and N_r denote the number of transmit and receive antennas, respectively, the received signal is described by

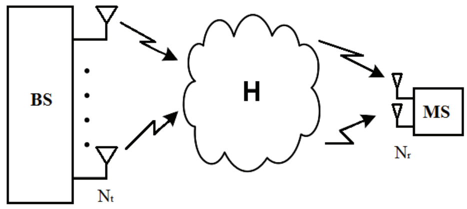

Fig. 1. Spatial Multiplexing Noise-Limited System.

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{x}$ is the N_t-dimensional transmit complex vector, each element of which is chosen from a constellation, $\mathbf{H}$ is the $N_r \times N_t$ complex, random channel matrix that presents the channel characteristics of a slow and flat fading environment and $\mathbf{n}$ is an N_r-dimensional complex additive white gaussian noise (AWGN) vector with covariance matrix $\Phi_{N_r} = \sigma^2\mathbf{I}$, where $\mathbf{I}$ represents the identity matrix.

The transmit vector $\mathbf{x}$ is constrained to have overall power given by

$$\frac{1}{2}E\left\{\mathbf{x}^\dagger\mathbf{x}\right\} \leq P \tag{2}$$

where $\frac{1}{2}E\left\{\mathbf{x}^\dagger\mathbf{x}\right\}$[1] represents the average power of the constellation and P is the total power which is constant for the purpose of comparison to non-SM system.

The entries of $\mathbf{H}$ are independent with uniformly distributed phase and Rayleigh distributed fading amplitude, modeling a Rayleigh slow and flat fading channel with sufficient physical separation between the N_t transmission and N_r reception antennas.

In a interference-limited environment as illustrated in Fig. 2, there will be N_t interfering signals, so the received signal is now given by

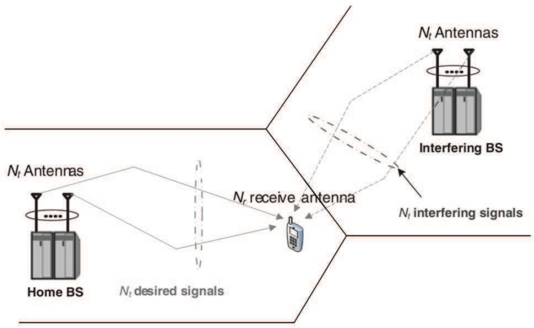

Fig. 2. Spatial Multiplexing Interfered-Limited System.

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[1] † denotes a conjugate transpose.

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \frac{\rho}{N_t}\mathbf{H_i}\mathbf{x_i} + \mathbf{n} \tag{3}$$

where $\mathbf{x_i}$ and $\mathbf{H_i}$ represent the transmit vector and the random channel matrix respectively for one dominant interferer, and ρ is an amplitude factor, which allow us to vary the signal-to-interference ratio (SIR).

Note that the interference is divided by N_t in order to maintain the total power fixed. Assuming that transmitted vectors $\mathbf{x}$ and $\mathbf{x_i}$ are synchronous, which represents the worst case [7], the SIR is defined as

$$SIR = \frac{P_{\mathbf{x}}}{\rho^2 P_{\mathbf{x_i}}} \tag{4}$$

where $P_{\mathbf{x}}$ and $P_{\mathbf{x_i}}$ are the total power of $\mathbf{x}$ and $\mathbf{x_i}$ transmitted vectors, respectively.

From (4) and with $P_{\mathbf{x}} = P_{\mathbf{x_i}}$, to the transmitted vectors are allocated the same transmission power P, according to (2). So, the SIR is reduced to

$$SIR = \frac{1}{\rho^2} \tag{5}$$

III. Optimum Decoding: Maximum Likelihood Decoder

The SM decoder technique used in this paper is the maximum likelihood decoder (MLD) [2], which finds the most likely input vector $\hat{\mathbf{x}}$ via a minimum-distance criterion.

Assuming that the receiver knows $\mathbf{H}$ (e.g. via transmitting training sequences), we choose $\hat{\mathbf{x}}$ as the transmitted vector that minimizes

$$\|\mathbf{r} - \mathbf{H}\hat{\mathbf{x}}\|^2 \tag{6}$$

Unfortunately, there is no simple way to compute this, and an exhaustive search must be done over all M^{N_t} possible input vectors, where M is the order of the modulation (e.g., M = 4 for QPSK). So, the complexity grows exponentially with N_t, which is the main disadvantage. For a small number of transmitting antennas ($N_t < 5$), however, the complexity is comparable to other decoders [2].

On the other hand, the MLD even works well when the number of N_t is larger than the number of N_r, which is not possible for conventional techniques. Hence, it is always possible to increase the data rate by increasing N_t. It seems somewhat surprising that it is possible to have more transmitters antennas than receivers antennas.

IV. Performance Analysis

Spatial multiplexing (SM) systems can be compared to single-antenna-transmission (non-SM) systems that employ high order modulations, thus, both systems present the same spectral efficiency.

In order to make a fair comparison, the transmission power of SM systems is normalized by a $1/N_t$ factor, thereby the both systems have also the same transmit power. Thus, the signal-to-noise ratio (SNR) per symbol in SM can be expressed as

$$\gamma_s = \frac{E_x}{N_t N_o} \tag{7}$$

where E_x represents the average energy of each transmitted symbol belonging to $\mathbf{x}$, and N_o represents the noise power spectral density.

The average probability of error can be used as a metric to evaluate the performance of a system in a noise-limited environment, and can be computed by integrating the error probability in an AWGN channel over the fading distribution

$$\overline{P_s} = \int_0^\infty P_s(\gamma_s)\, p_{\Gamma_s}(\gamma_s)\, d\gamma \tag{8}$$

where $P_s(\gamma_s)$ is the symbol error probability in AWGN channel with SNR per symbol given in (7). With fading, the SNR can be written as $\gamma_s = \alpha^2 E_x / N_t N_o$, and represents the instantaneous SNR per symbol, where α is a Rayleigh random variable (RV) that represents the fading with probability density function (PDF) given by

$$p_A(\alpha) = \frac{\alpha}{\sigma^2} e^{-\alpha^2/2\sigma^2}, \qquad \alpha \geq 0 \tag{9}$$

where $\sigma^2 = E\left\{\alpha^2\right\}/2$, where $E\{\cdot\}$ denotes the expectation operator.

From (9), we can compute $p_{\Gamma_s}(\gamma_s)$ by making the change of variable [8]

$$p_{\Gamma_s}(\gamma_s) d_{\gamma_s} = p_A(\alpha) d_\alpha \tag{10}$$

and we get

$$p_{\Gamma_s}(\gamma_s) = \frac{1}{\overline{\gamma_s}} e^{-\gamma_s/\overline{\gamma_s}}, \qquad \gamma_s \geq 0 \tag{11}$$

where $\overline{\gamma_s}$ represents the average SNR per symbol in SM.

Finally, if Gray encoding is used, the BER is given by

$$P_b \approx \frac{P_s}{\log_2 M} \tag{12}$$

A. Co-channel Interference

In the presence of CCI (interference-limited environment), the error probability in AWGN channel is $P_s(\gamma_s, \gamma_i)$, where γ_s and γ_i represent the instantaneous SNR per symbol of the desired and interferer signal, respectively.

From (8), the average symbol error probability with fading and CCI is given by

$$P_{s,i} = \int_0^\infty \int_0^\infty P_s(\gamma_s, \gamma_i)\, p_{\Gamma_s}(\gamma_s)\, p_{\Gamma_i}(\gamma_i)\, d\gamma_s d\gamma_i \tag{13}$$

where $p_{\Gamma_s}(\gamma_s)$ and $p_{\Gamma_i}(\gamma_i)$ are the PDFs as (11) of γ_s and γ_i, respectively.

B. M-ASK

The symbol error probability for a M-ASK modulation scheme in an AWGN channel without CCI is given by [9]

$$P_s = \frac{2\,(M-1)}{M} Q\left(\sqrt{6\frac{E_b}{N_o}\frac{\log_2 M}{(M^2-1)}}\right) \tag{14}$$

In the presence of one dominant interferer, the symbol error probability is given by [10]

$$P_{s,M-ASK}\left(\gamma_s,\gamma_i\right) = \frac{2(M-1)}{M^2}\sum_{m=0}^{\frac{M}{2}-1}\sum_{k=0}^{1} Q(A) \tag{15}$$

where $A = \left[\sqrt{\gamma_s} - (2m+1)(1-2k)\rho\sqrt{\gamma_i}\right]\sqrt{6\frac{\log_2 M}{M^2-1}}$

C. M-QAM

The symbol error probability for a M-QAM modulation can be obtained from the cartesian product between two $\sqrt{M}$-ASK signals [9]

$$P_{s,M-QAM} = 1 - \left(1 - P_{s,\sqrt{M}-ASK}\right)^2 \tag{16}$$

where $P_{s,M-ASK}$ is the symbol error probability of $\sqrt{M}$-ASK given by

$$P_{s,\sqrt{M}-ASK}\left(\gamma_s,\gamma_i\right) = \frac{2\left(\sqrt{M}-1\right)}{M}\sum_{m=0}^{\frac{\sqrt{M}}{2}-1}\sum_{k=0}^{1} Q(B) \tag{17}$$

where $B = \left[\sqrt{\gamma_s} - (2m+1)(1-2k)\rho\sqrt{\gamma_i}\right]\sqrt{3\frac{\log_2 M}{M-1}}$

V. SIMULATION RESULTS

In this section, we present the results in terms of the average BER for a cellular network downlink in the presence of one dominant interferer, where due to space and cost restrictions on the mobile unit, the number of receiving antennas is N_r = 1. But, before that, in a noise-limited environment, we formulate an interesting question: for a given data rate, is it better to transmit at low power with multiple transmit antennas or with full power with only one antenna at full rate, using some higher order QAM modulation in order to maintain the same spectral efficiency?. Fig. 3 shows some plots to give an answer to this problem.

Three systems are simulated that transmit 4 bits per symbol, ranging from BPSK symbols on 4 transmit antennas, passing to QPSK symbols on 2 transmit antennas and finally a 16-QAM symbol on 1 transmit antenna. It can be seen that 2 transmit antennas using QPSK is the best choice with an SNR advantage of about 3 and 1 dB over the BPSK and 16-QAM systems, respectively.

Fig. 4 shows another comparison between three systems with same spectral efficiency, in this case 8 bits per symbol. Again, the system using QPSK on 4 transmit antennas is the best, with an SNR advantage of about 1 dB and 4 dB over the

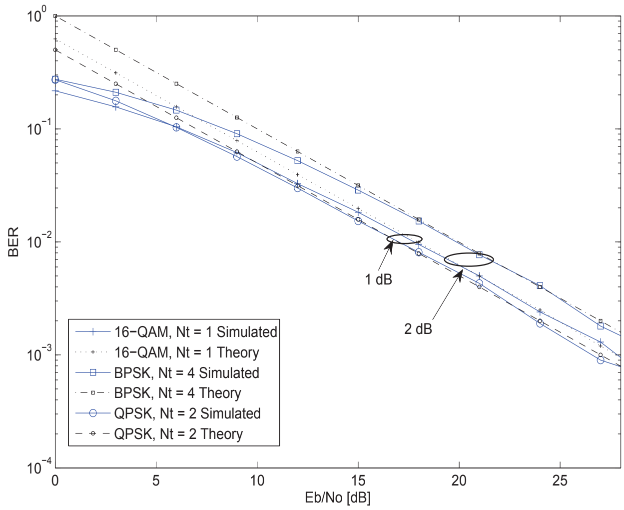

Fig. 3. BER as a function of E_b/N_o for BPSK with $N_t = 4$, QPSK with $N_t = 2$ and 16-QAM with $N_t = 1$.

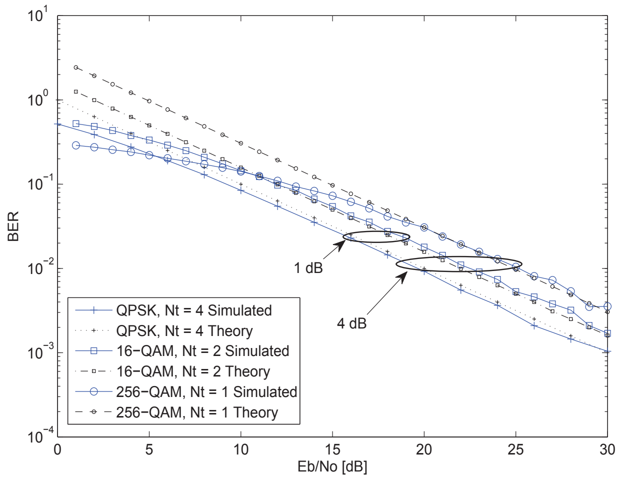

Fig. 4. BER as a function of E_b/N_o for QPSK with $N_t = 4$, 16-QAM with $N_t = 2$ and 256-QAM with $N_t = 1$.

system with 16-QAM on 2 transmit antennas and 256-QAM on 1 antenna system.

Fig. 5 shows the performance of QPSK systems where the number of transmit antennas varies from 1 to 4. The increased data rates cost an increase in E_b/N_o, where we need 6 dB increase by going from 1 to 4 antennas.

From the above results, QPSK modulation in a SM system is the one with best performance in terms of the average BER, for this reason, we analyze this modulation in the presence of co-channel interference (interference-limited environment).

Fig. 6 presents the BER as a function of E_b/N_o with one dominant co-channel interferer for a QPSK system with 2 transmit antennas and a single antenna 16-QAM system for *SIR* = 0, 9, 12, 24, 48 dB. For *SIR* = 0 dB, we observe that BER floor approaches 1/4 and 3/10 for QPSK and 16-QAM systems, respectively. This is due to the fact that the interference power is equal to the signal power and there is no

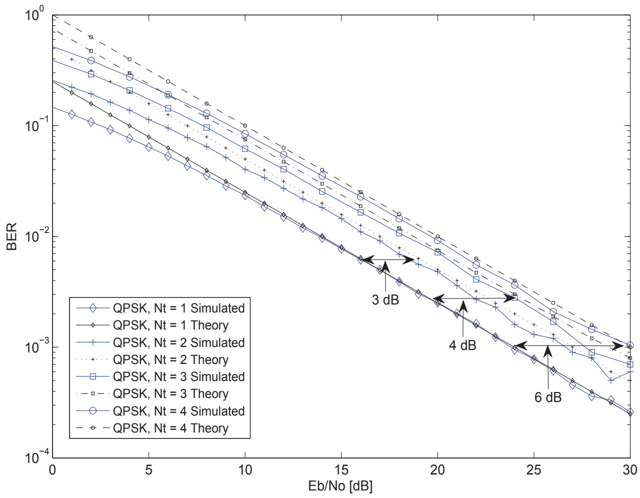

Fig. 5. BER as a function of E_b/N_o for QPSK with $N_t = 1$, $N_t = 2$, $N_t = 3$, and $N_t = 4$.

significant difference between the two systems. For *SIR* values in the range of 9 to 24, we have a significant degradation in the BER due to the effects of co-channel interference presenting floor regardless of any E_b/N_o increasing. We notice that the QPSK system has a better performance in terms of BER that the 16-QAM system in the presence of CCI. For *SIR* = 48 dB, the interference power is very small and can be negligible, so the performance of the BER corresponding to a noise-limited environment, and in this case the SNR advantage of about 1 dB between QPSK and 16-QAM system is maintained.

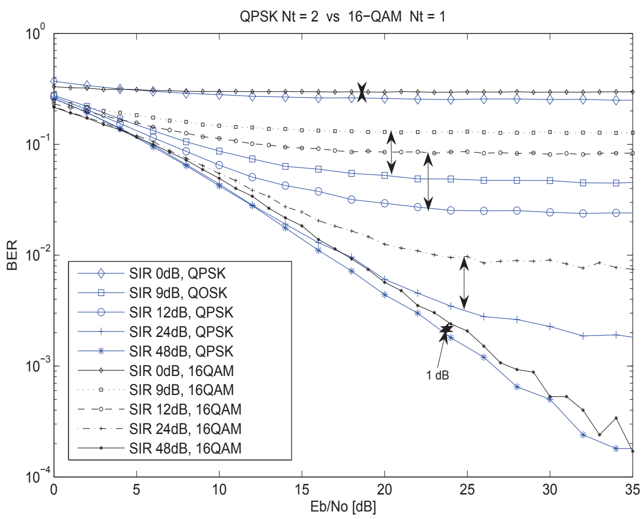

Fig. 6. BER as a function of E_b/N_o and SIR, for QPSK with $N_t = 2$ and 16-QAM with $N_t = 1$ in the presence of one CCI.

Fig. 7 and Fig. 8 show the BER as a function of E_b/N_o and *SIR* for QPSK with 3 transmit antennas versus a single-antenna 64-QAM system, and QPSK with 4 transmit antennas versus a single-antenna 256-QAM system, respectively. The curves and conclusions are similar to the previous case.

In a spatial multiplexing system increasing the number of transmitting antennas N_t and using QPSK modulation, we get

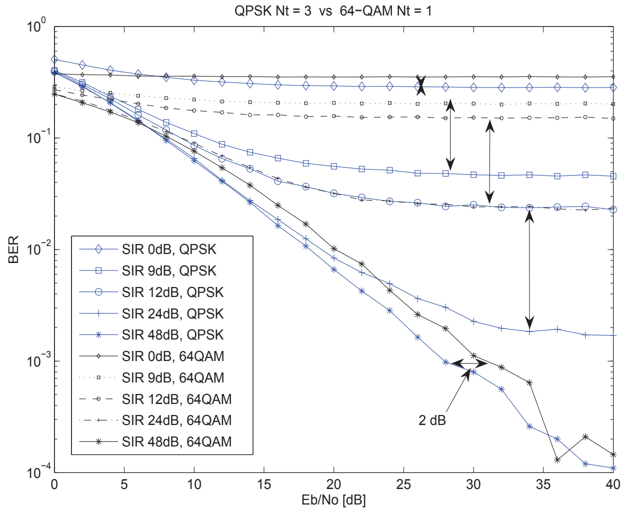

Fig. 7. BER as a function of E_b/N_o and SIR, for QPSK with $N_t = 3$ and 64-QAM with $N_t = 1$ in the presence of one CCI.

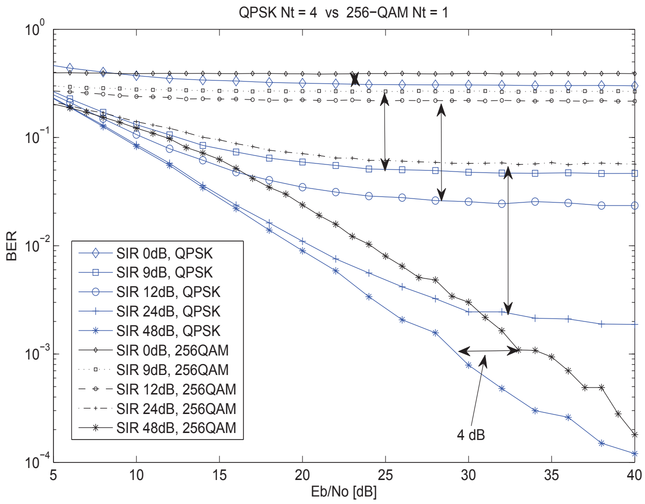

Fig. 8. BER as a function of E_b/N_o and SIR, for QPSK with $N_t = 4$ and 256-QAM with $N_t = 1$ in the presence of one CCI.

a better BER performance than a single-antenna *M*-QAM.

VI. Conclusions

In this paper, we have presented an analysis through simulations of the BER for SM systems in a noise-limited and interference-limited environment. The results were compared to a single-antenna systems (non-SM) with same transmit power and spectral efficiency.

In a noise-limited environment, a SM system which uses QPSK modulation presents better performance in terms of BER than a single-antenna system with high order modulation *M*-QAM. On the other hand, the presence of CCI causes significant degradation in the performance of both systems, presenting floor in the BER curves. We would like to emphasizing that a SM system with QPSK modulation presents less degradation compared to a single-antenna system with high order modulation *M*-QAM.

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